



# Projecting the Mortality Experience of Canadian Insured Lives

Seminar for the Appointed Actuary

PD 8

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Waterloo

# Research Team

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- NB – this is work in progress!

# Introduction

- Primary goal:  
*“Development of Mortality Improvement Scale (or Scales) to Incorporate Mortality Improvement into Product Analysis and Reserving for All Appropriate Products of Canadian Life Insurance Companies”*  
but we are only looking at insured lives
- Available data:
  1. Experience from individual life insurance
    - a) 1992-1993 to 2001-2002 (10-year period)
    - b) Categorized by sex, and by smoker status
    - c) IIPR, U - Waterloo

# Introduction

## 2. Experience of the Canadian Population

- a) 1921 to 2002 (82-year period)
- b) Sex-specific
- c) Provided by the Human Mortality Database and the CANSIM, Statistics Canada

# Introduction

- Idea:
  1. Project the mortality experience of the Canadian Population
  2. Relate the mortality experience of Canadian insured lives to that of the Canadian population
  3. Based on this relationship, project the insured life experience
  4. Best estimate and confidence intervals for mortality projection

# Outline 1 – Population Data

- An overview of the original Lee-Carter model
  - a) Model Structure
  - b) Parameter estimation
  - c) Computation of interval forecast
  - d) Problems of the original model
- A new variant of the model

# Outline 2 –Insured Lives Data

- An overview of the available data
- Graduation of assured lives' mortality experience
- Assured lives vs. population
  1. Graphical comparison
  2. Mathematical Relationship
- Reduction Factors
  1. VBT (SoA Task Force)
  2. CMIB
  3. Lee-Carter
- Confidence Intervals

# The Lee-Carter Model

$$\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}$$

- $m_{x,t}$  is the central rate of death at age  $x$  at time  $t$
- $a_x$  is the average level of mortality age  $x$
- $b_x$  is the relative speed of change at each age
- $k_t$  is an index of the level of mortality at time  $t$
- $\varepsilon_{x,t}$  is the error term

# The Lee-Carter Model

- The series  $k_t$  is obtained when the model is fitted to historical data
- $k_t$  is then projected to the future by a univariate time-series model (ARIMA)
- Usually, a random walk with drift gives a good fit

$$k_t = c + k_{t-1} + \xi_{x,t}$$

# Parameter Estimation

- **Two distinct methods**
  - Singular Value Decomposition (SVD)
    - E.g. Lee and Carter (1992,2000), Li and Chan (2004) x 2
  - Maximum Likelihood Estimation (MLE)
    - E.g. Renshaw and Haberman (2003) X 3, Wilmoth (1993, 2000)
- Data:  $(D_{x,t}, N_{x,t})$  = deaths and central exposed to risk.

# SVD

- For Confidence Intervals:
  1. Assume independence between parameters:
  2. Assume normal residual terms for symmetric 95% CI:

$$\ln(m_{x,t}) = \hat{\ln}(m_{x,t}) \pm 2\sigma_{E,t+s}$$

# MLE

- Parameter estimation
  1. Assume

$$D_{x,t} \sim Poi(N_{x,t} \exp(a_x + b_x k_t))$$

2. Hence, we obtain the likelihood function

$$L(\mathbf{a}, \mathbf{b}, \mathbf{k}) = \prod_{x,t} \frac{(N_{x,t} \exp(a_x + b_x k_t))^{D_{x,t}} \exp(-N_{x,t} \exp(a_x + b_x k_t))}{D_{x,t}!}$$

# MLE

- Interval forecast:
  - By Bootstrapping the model (Brouhns et al., 2005, SAJ):

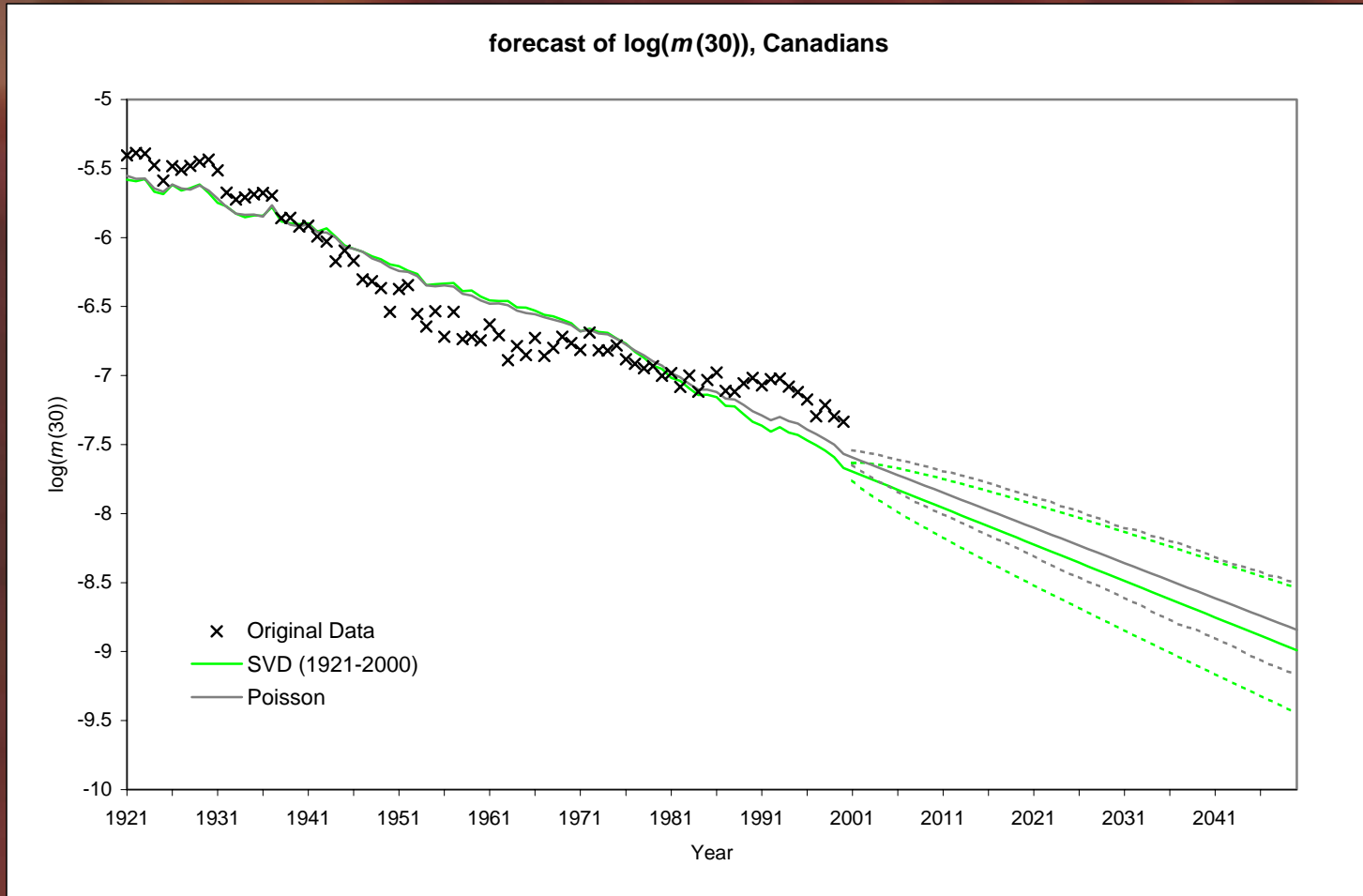
1. Simulate  $n$  realizations from

$$Poi\left(N_{x,t} \exp\left(\hat{a}_x + \hat{b}_x \hat{k}_t\right)\right)$$

# MLE

2. For each of the bootstrap samples:
  - a) Estimate the model parameters  $a$ ,  $b$ , and  $k$
  - b) Simulate the future values of  $k$  under the specified time-series model
  - c) Compute future death rates
3. Obtain an empirical distribution of future death rates, and the confidence interval are given by the percentiles

# Problems



# Problems

- The interval forecasts given by both approaches are **too narrow**
- SVD
  1. Variance of forecast error is underestimated
  2. Only symmetric intervals are allowed
  3. Independence of parameters may be questionable
- MLE
  1. Under Poisson density: Variance = Mean
  2. MLE is known to be not robust

# A New Variant

- We introduce a more flexible density – the negative binomial

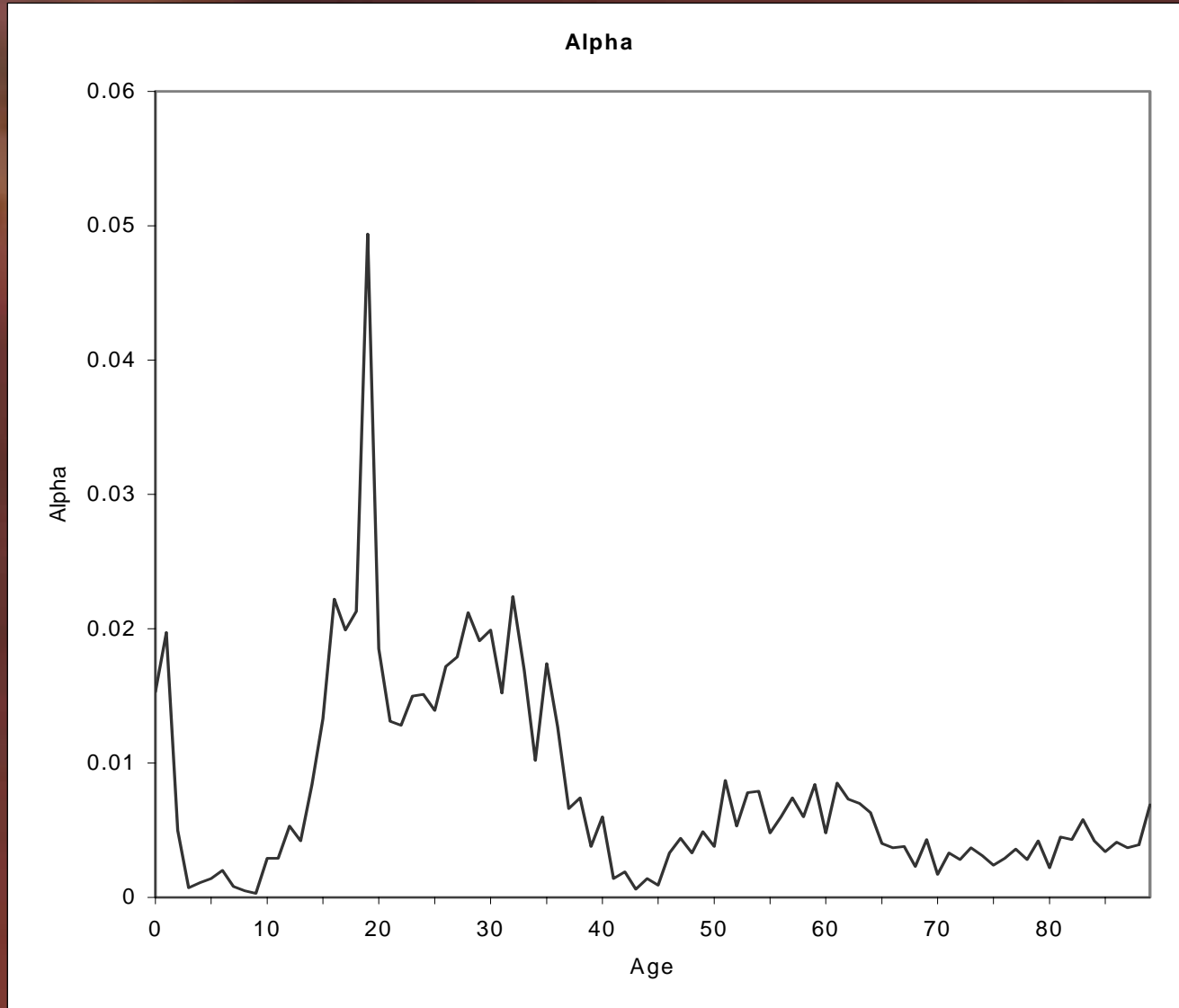
$$P(Y = y) = \binom{r + y - 1}{y} \left( \frac{1}{1 + \beta} \right)^r \left( \frac{\beta}{1 + \beta} \right)^y$$

Or

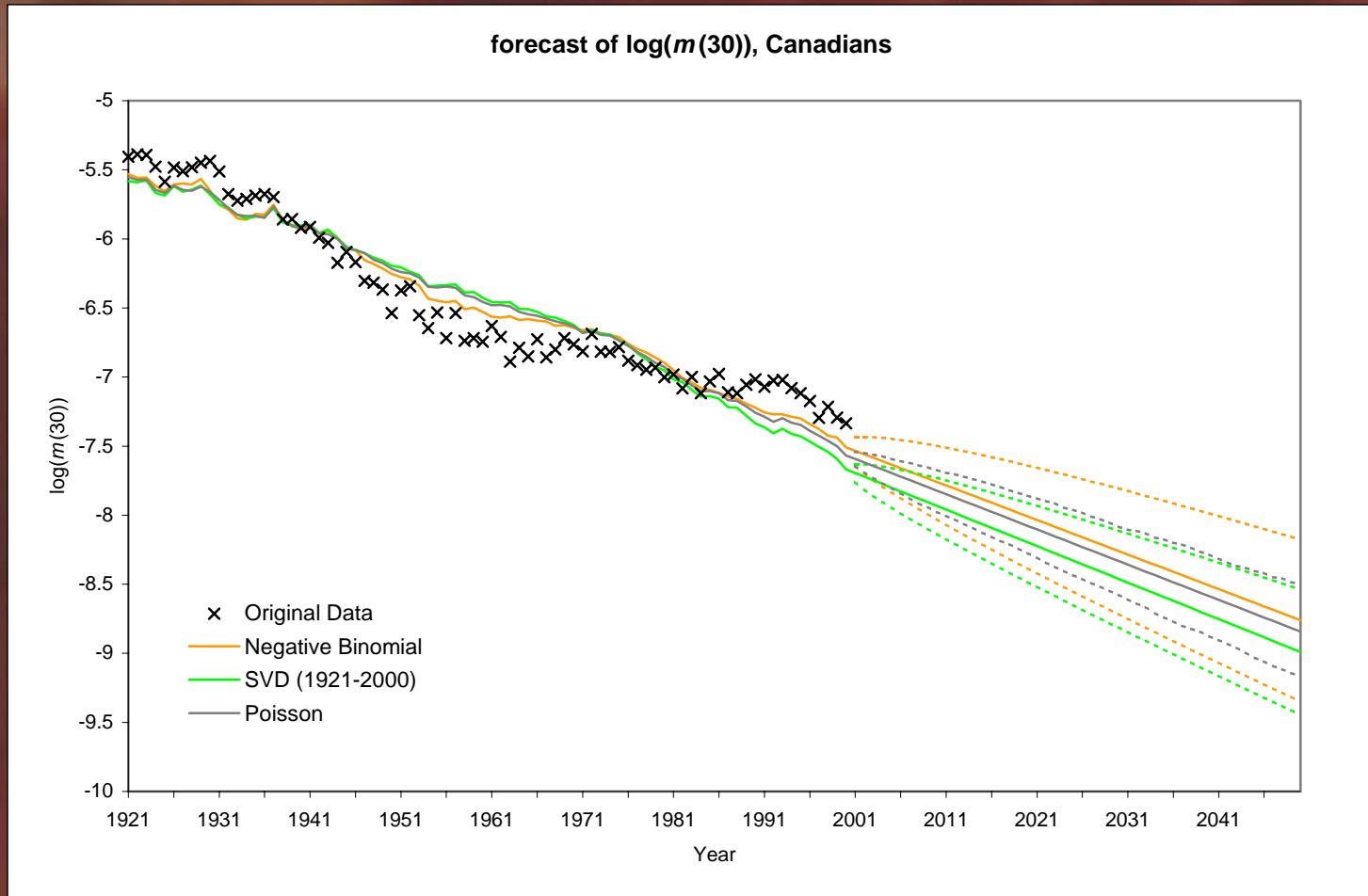
$$P(Y = y) = \frac{\Gamma(y + \alpha^{-1})}{y! \Gamma(\alpha^{-1})} \left( \frac{\alpha\mu}{1 + \alpha\mu} \right)^y (1 + \alpha\mu)^{-1/\alpha}$$

- Under Negative Binomial: Variance > Mean
- $a = 0$  yields Poisson

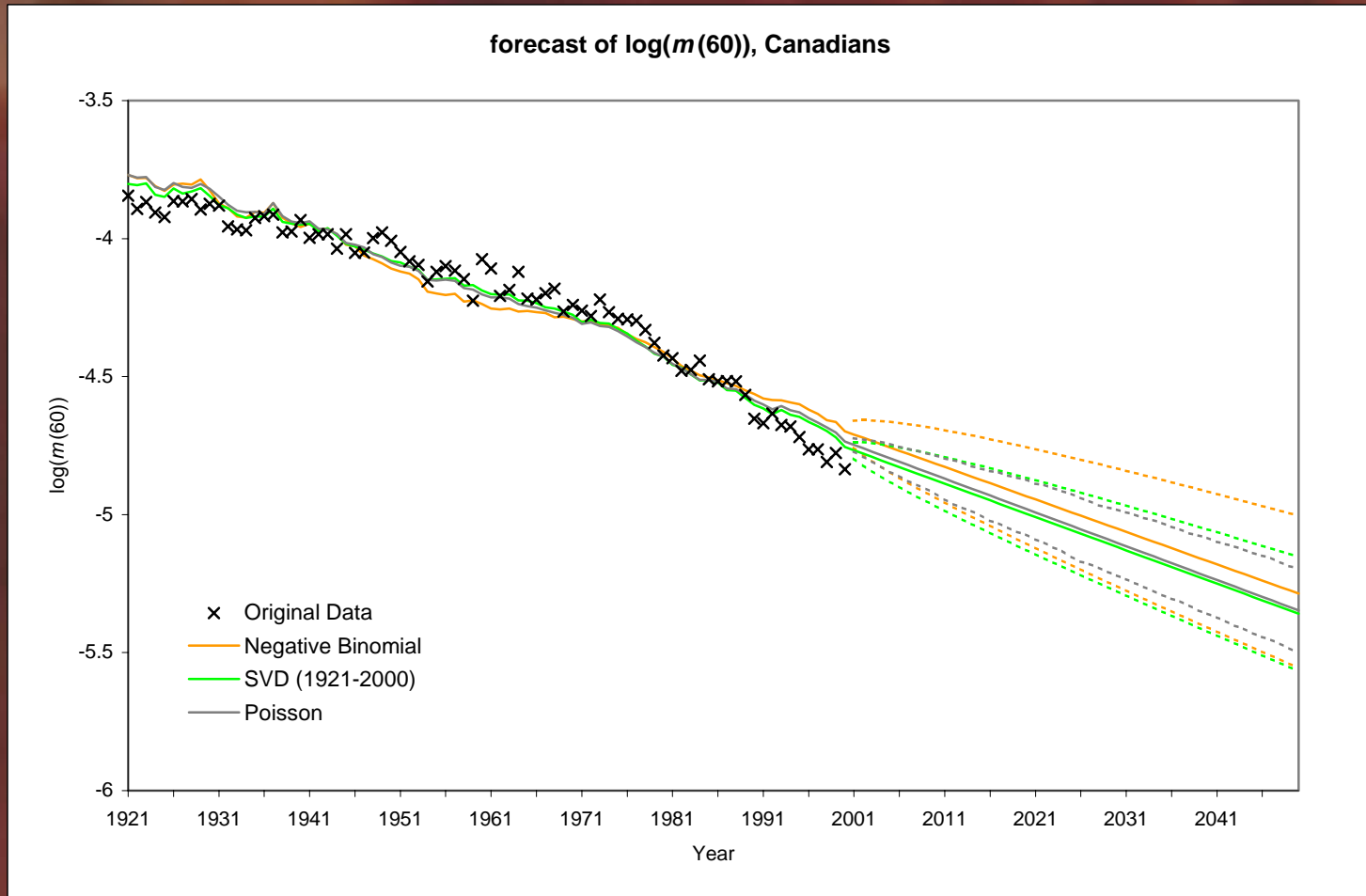
# Empirical Results



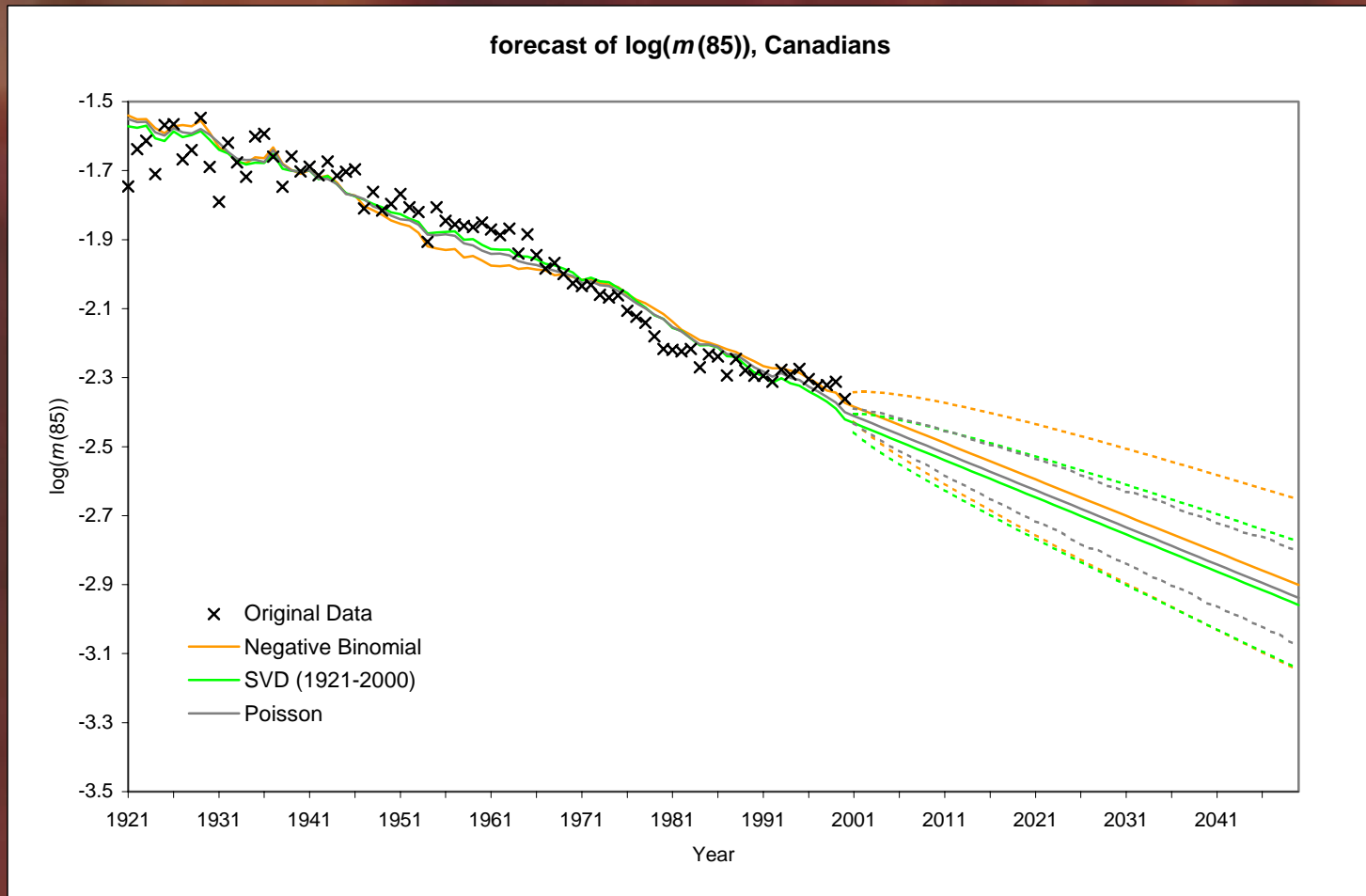
# Empirical Results



# Empirical Results



# Empirical Results



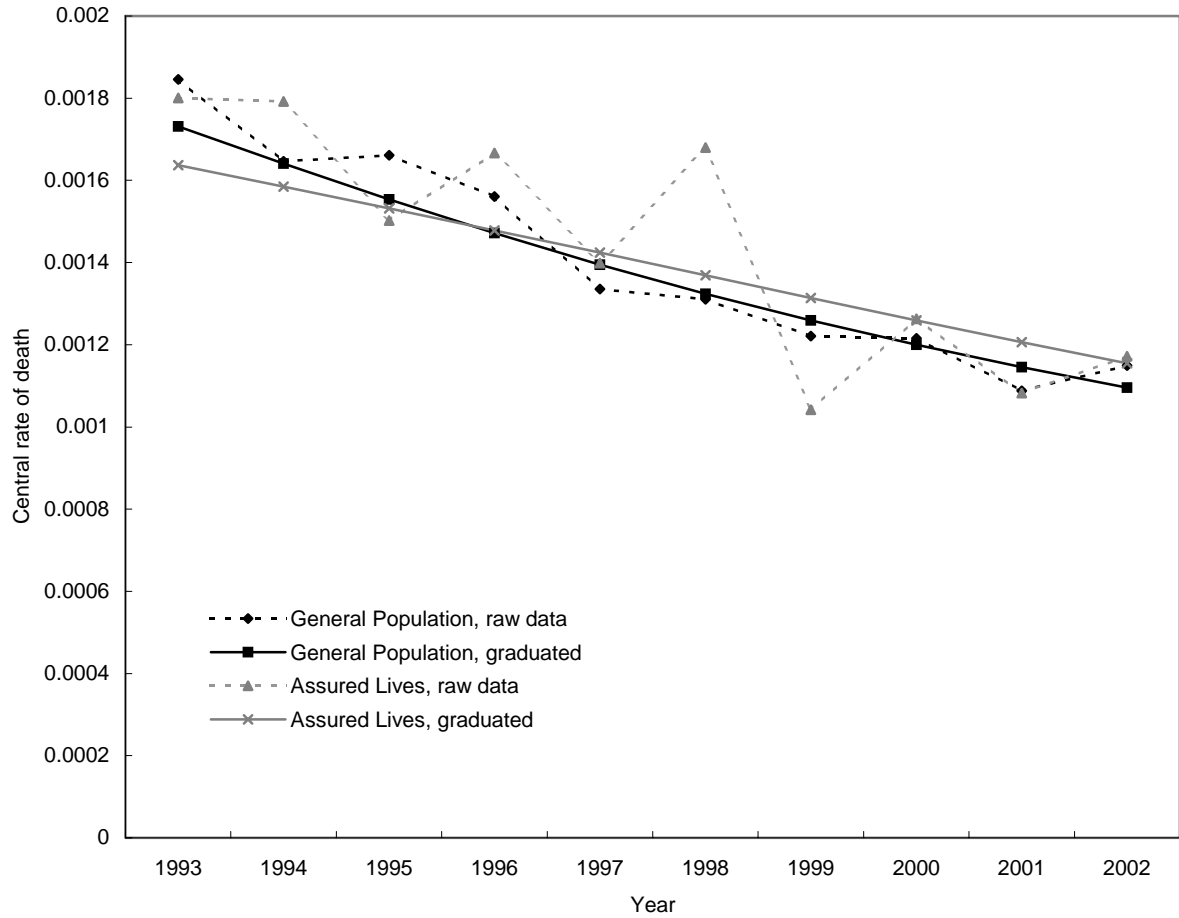
# The Insured Lives Data

- Classifications:
  - By Sex: Male, Female, indeterminate
  - By smoker-status: Smoker, non-smoker, not-reported
  - Problem (an example):
    - Exposure-to-risk at age 34, male, ultimate:
      - Smoker: 935
      - Non-smoker: 1515
      - Indeterminate: 11099
- Selection effect:
  - 15-year select period
- Available by individual age up to age 99

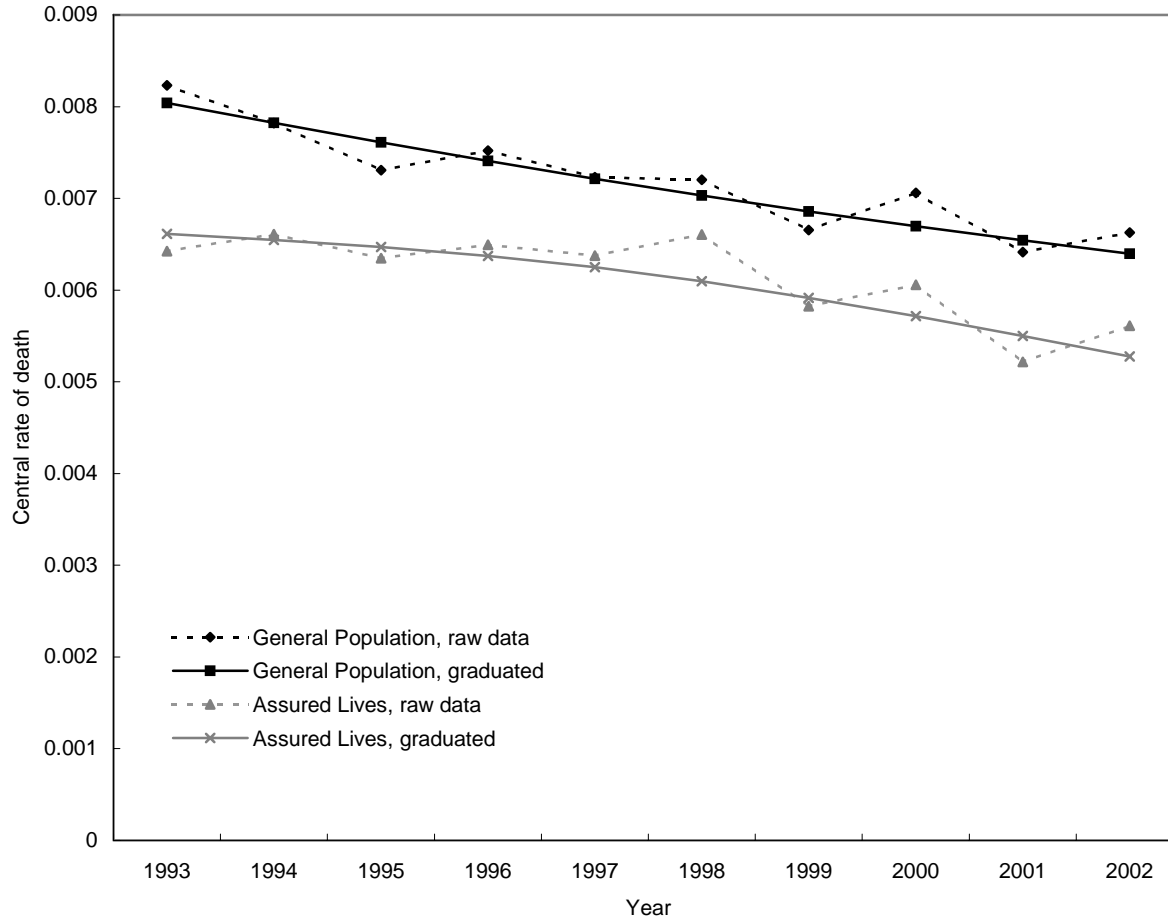
# Graduation of Assured Lives' Mortality Experience

- Graduation is required for two reasons:
  1. To obtain a reliable base-table to which the mortality reduction factors are applied
  2. To depict the trend underlying the assured life mortality experience
- Two-dimensional graduation is preferred
  - Smoothed over time and age
- 2-D  $P$ -Spline smoothing is used
  - Not Lee- Carter

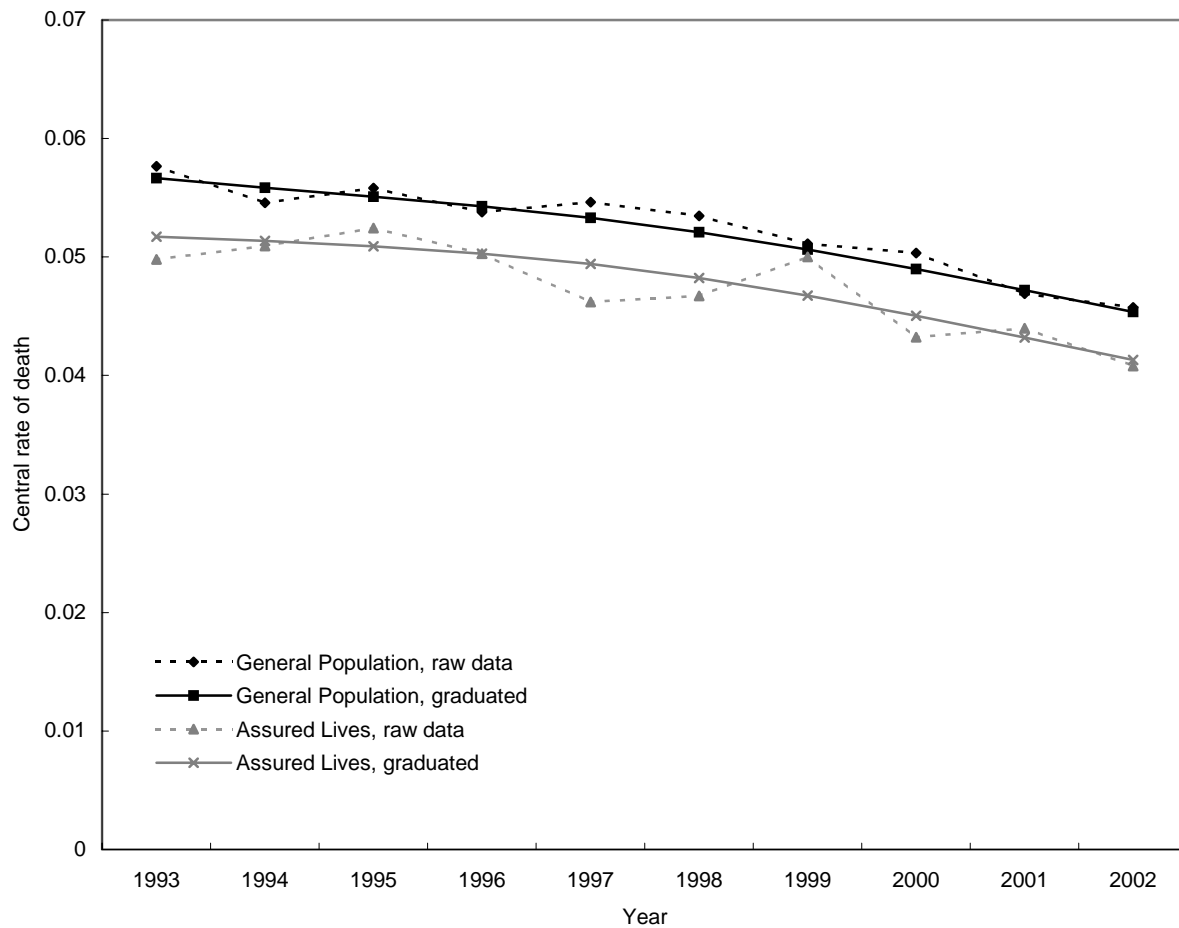
### Mortality Improvement, Age 36, Male



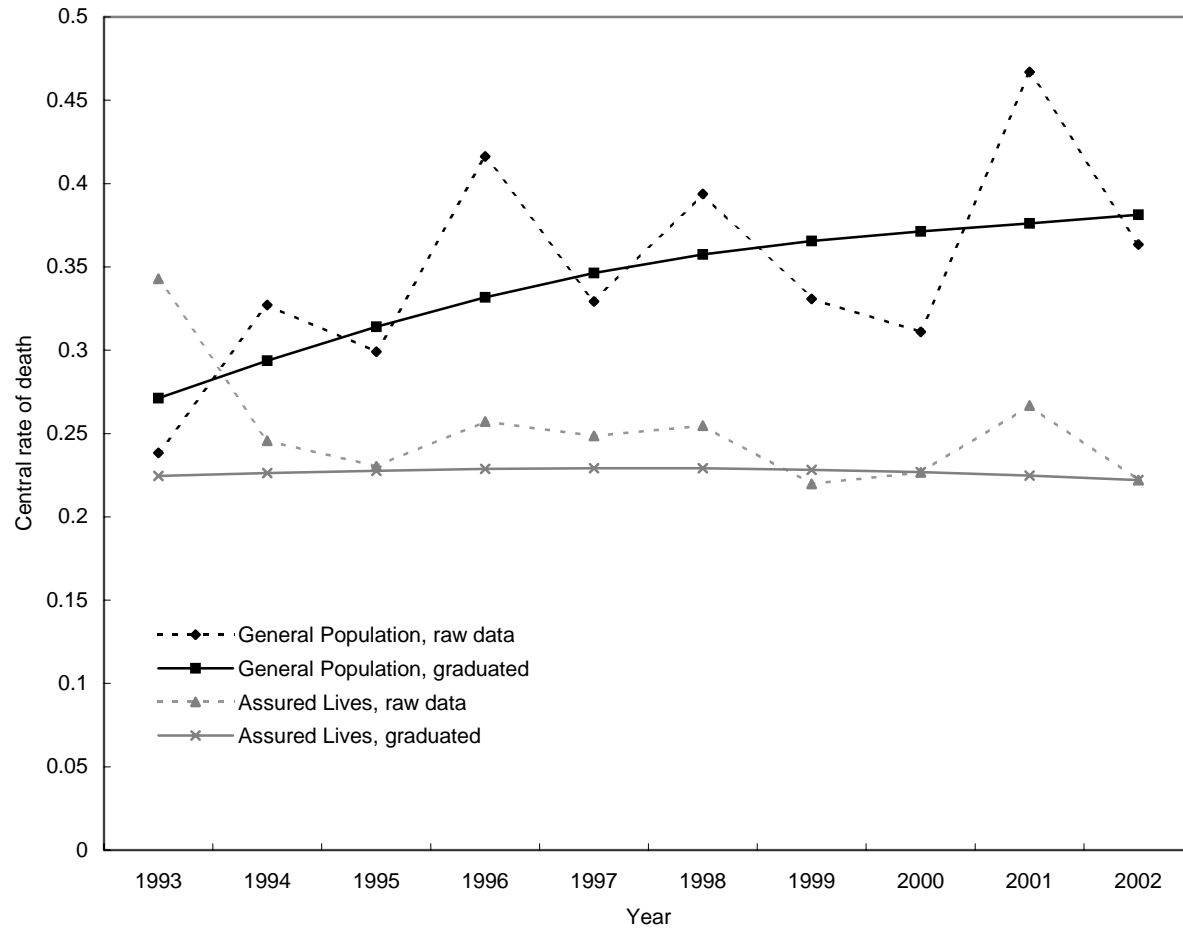
### Mortality Improvement, Age 56, Male



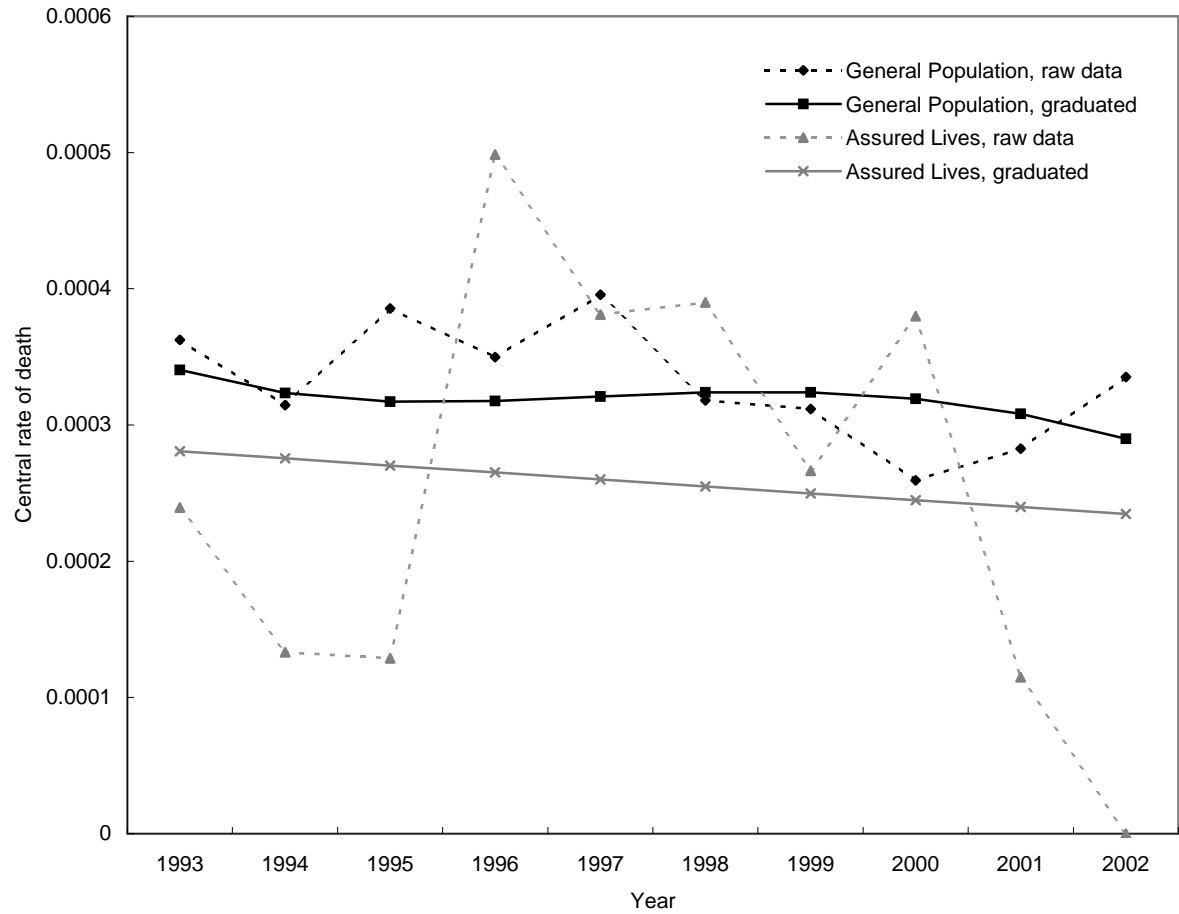
### Mortality Improvement, Age 76, Male



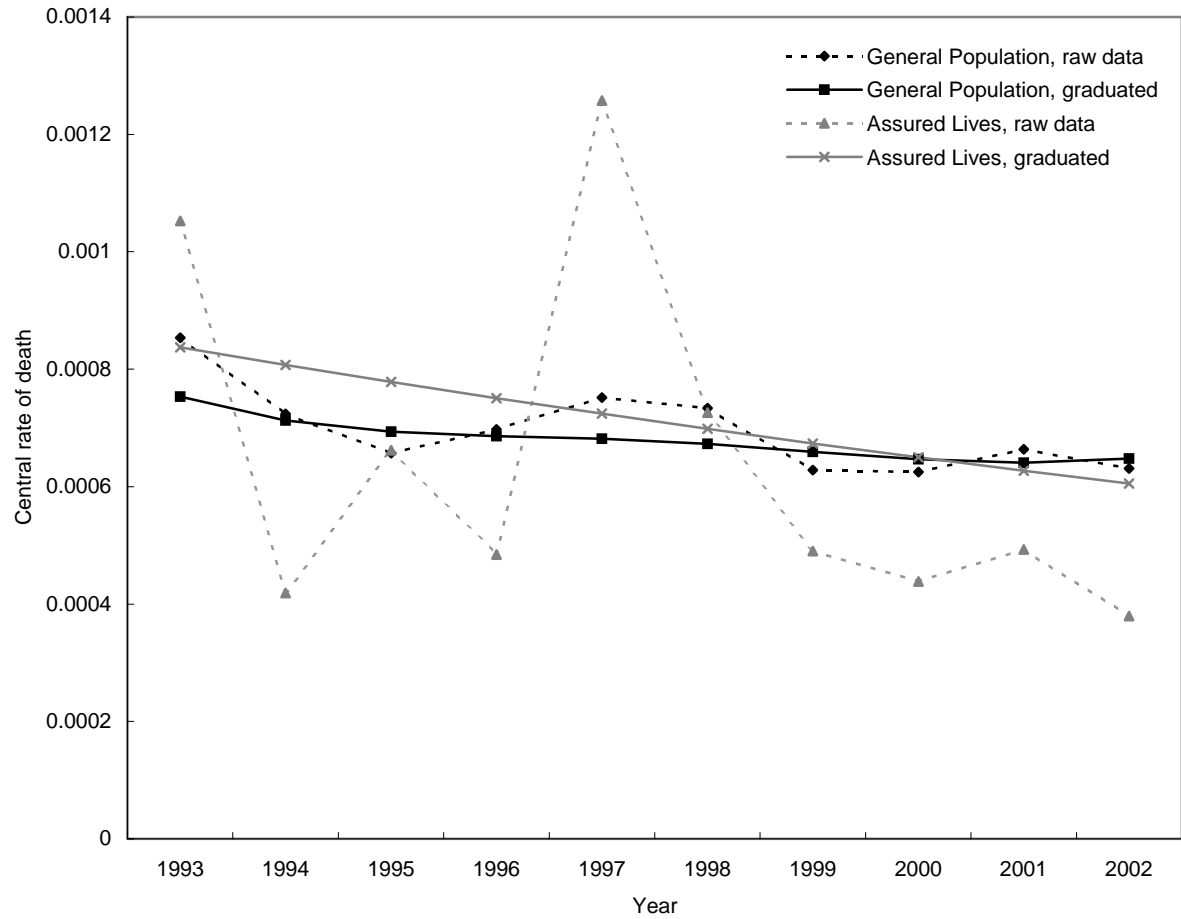
### Mortality Improvement, Age 96, Male



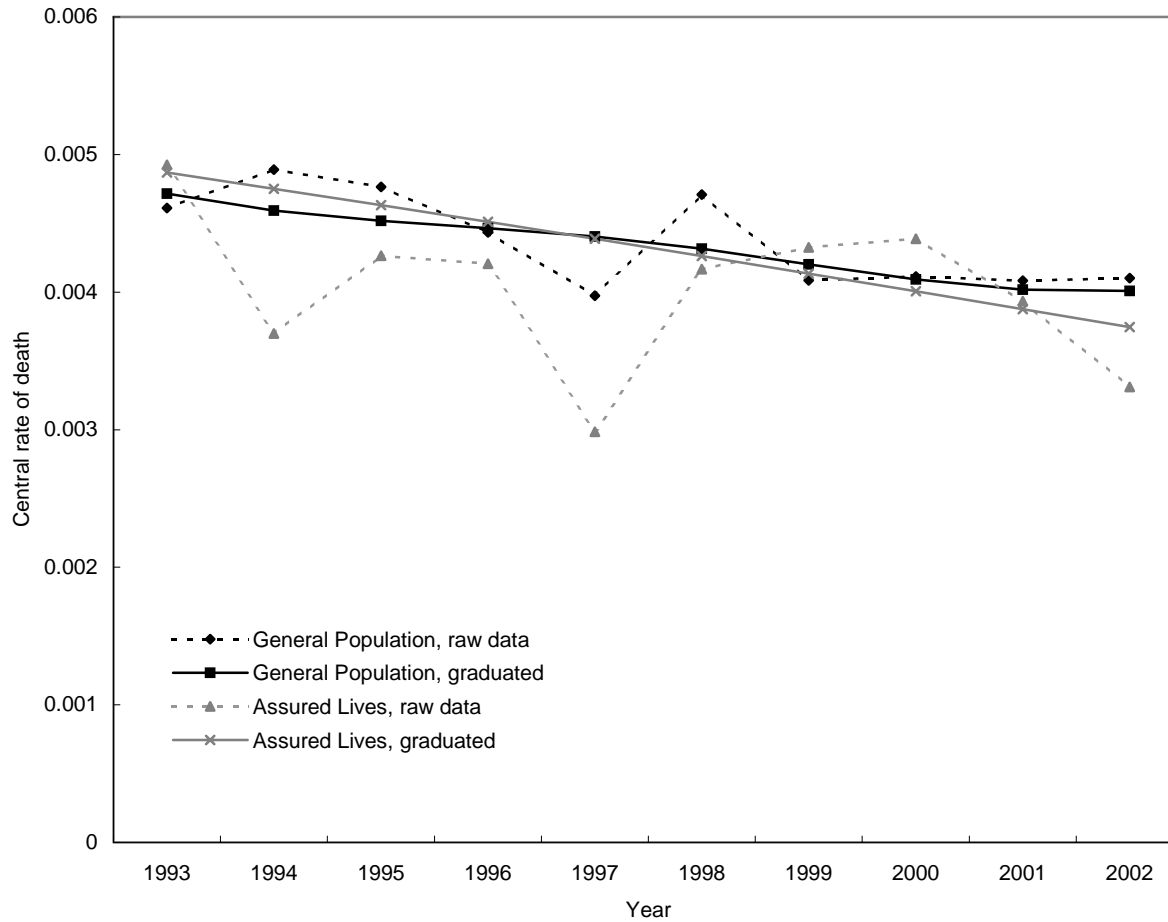
### Mortality Improvement, Age 16, Female



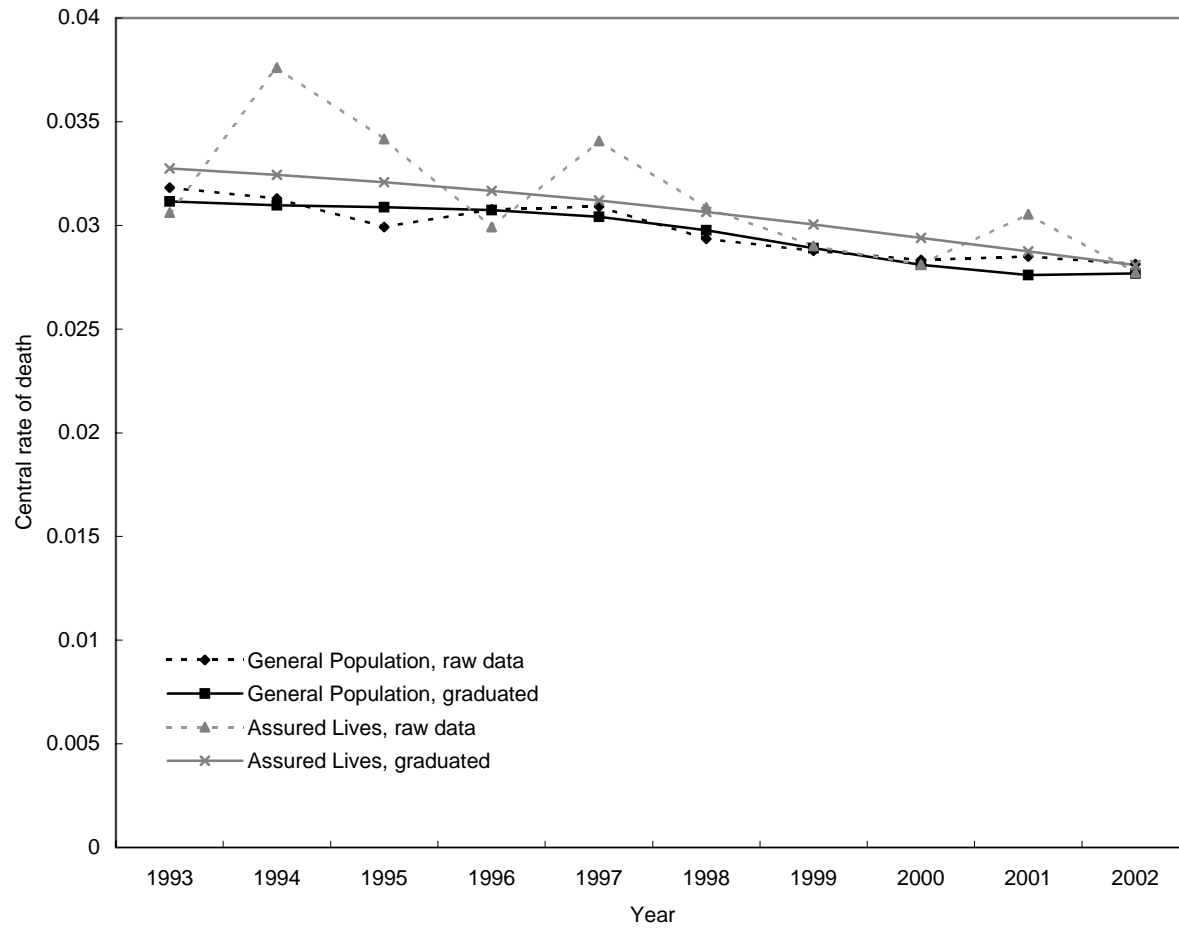
### Mortality Improvement, Age 36, Female



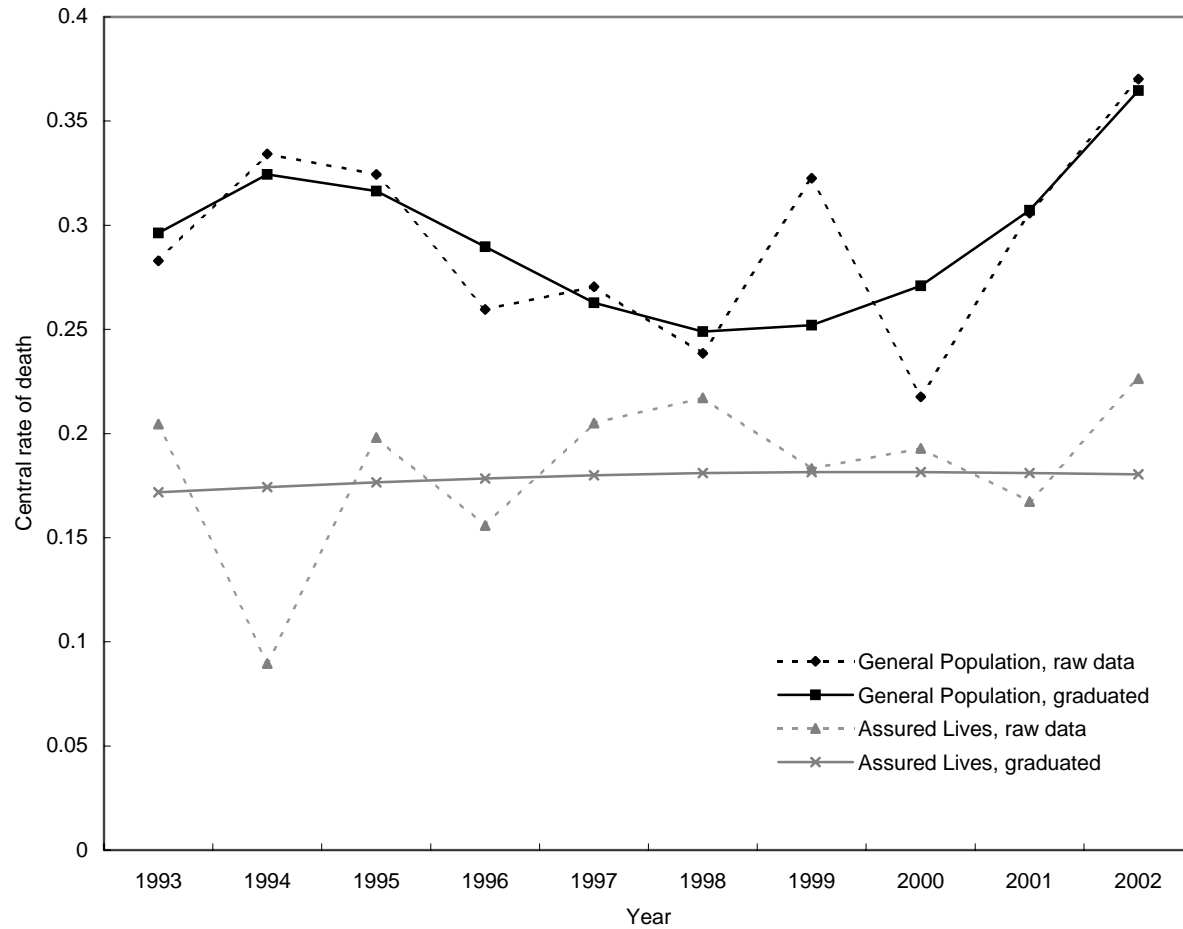
Mortality Improvement, Age 56, Female



### Mortality Improvement, Age 76, Female



### Mortality Improvement, Age 96, Male



# Assured Lives Vs. Population

- The patterns of mortality improvement differ
- The “Brass-type” mortality model may be used to relate the two mortality experiences
- Applications of “Brass-type”:
  1. Projecting old-age mortality in developed countries (United Nations, 1997)
  2. Estimation of the Belgian annuitants’ life table (Brouhns et al., 2002)

# Assured Lives Vs. Population

- The “Brass-type” model:

$$f(m_{x,t}) = \gamma_x + \eta_x f(m_{x,t}^{ref})$$

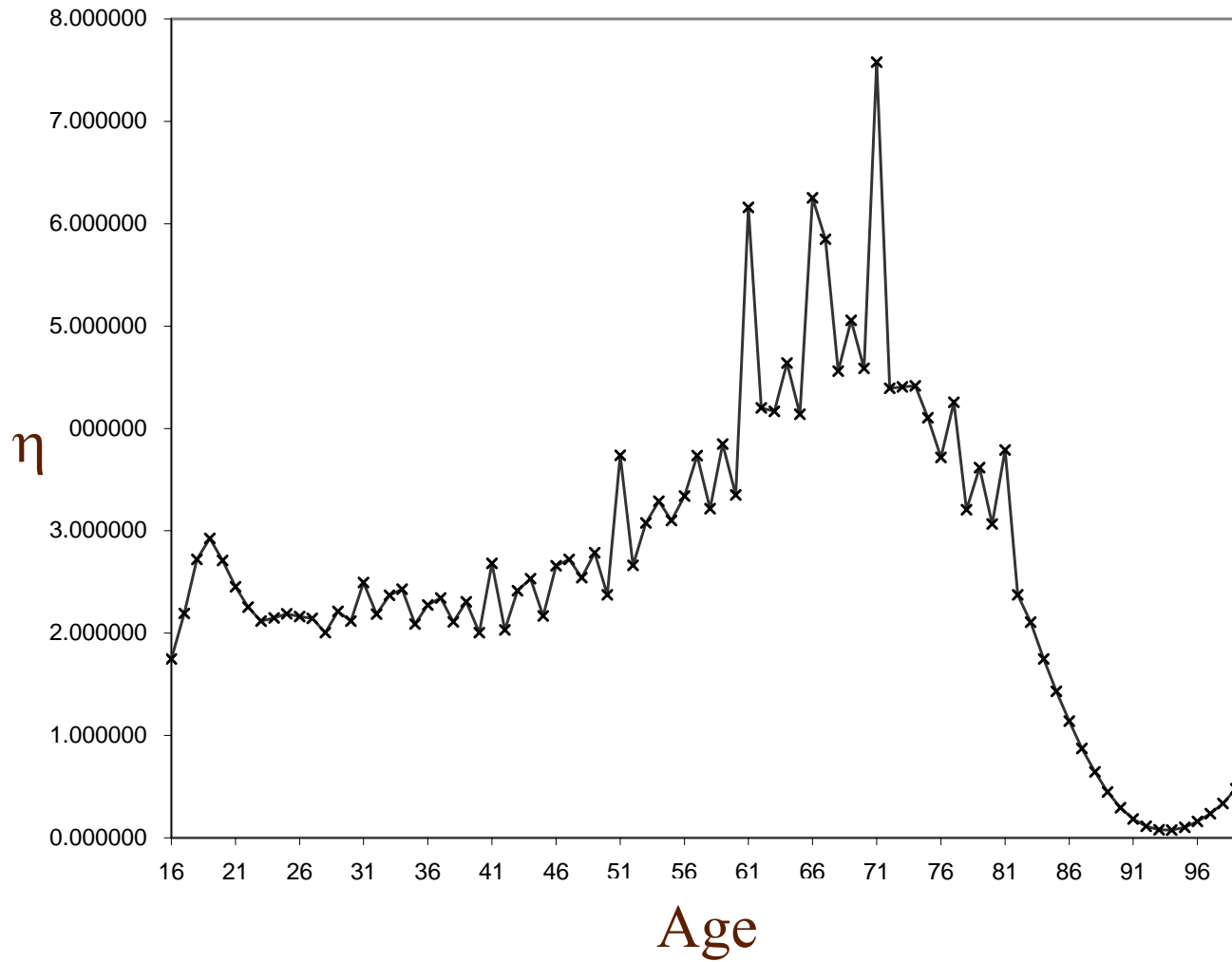
- Usually,  $f(y) = \ln(y)$ , or  $f(y) = \text{logit}(y)$

- We set  $\ln(m_{x,t}^{assured}) = \gamma_x + \eta_x \ln(m_{x,t}^{population})$

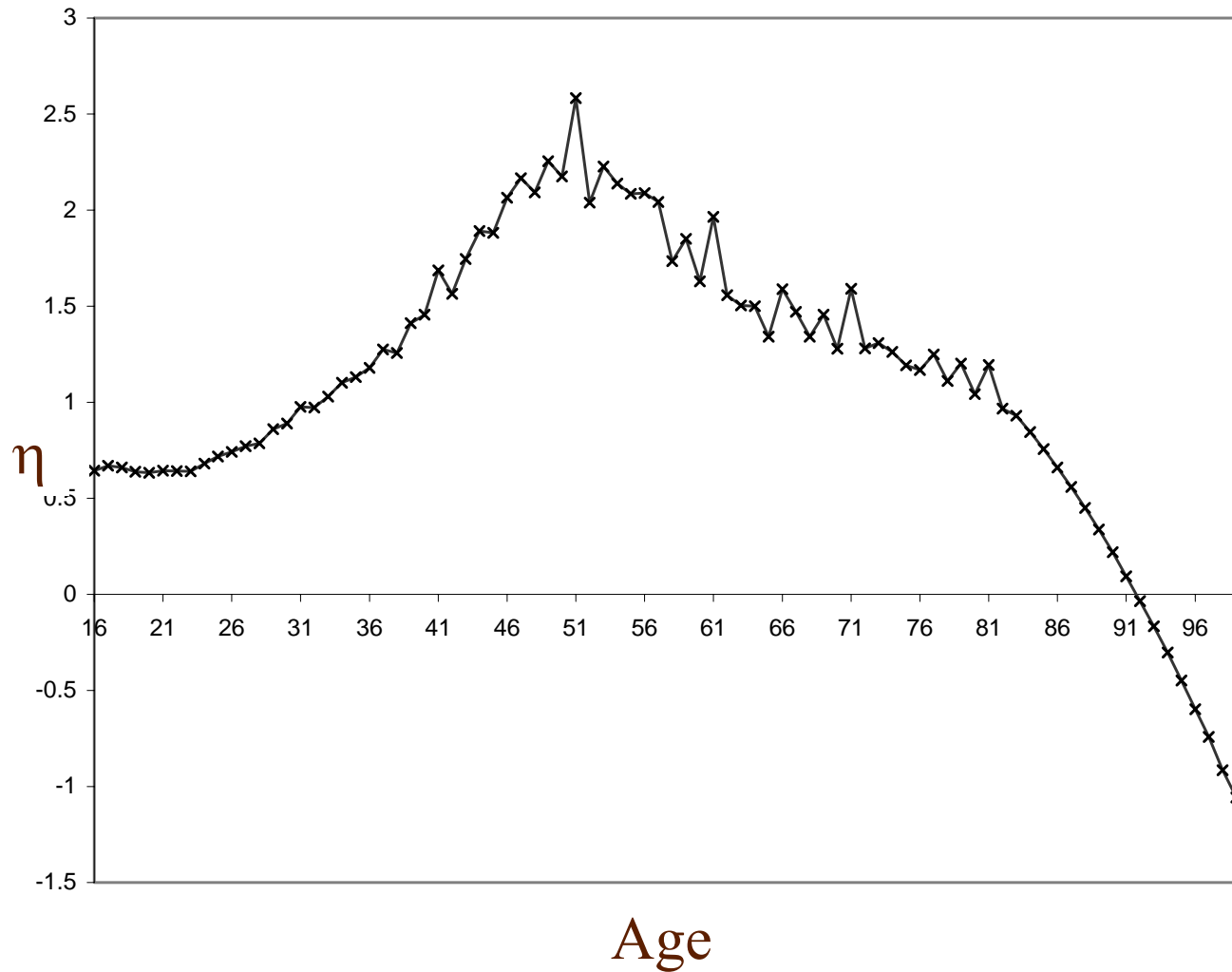
# Assured Lives Vs. Population

- Interpretation of  $\eta$ :
  - $\eta > 1$ 
    - Mortality of assured lives is improving faster
  - $1 > \eta > 0$ 
    - Mortality of assured lives is improving slower
  - $\eta = 0$ 
    - Mortality of assured lives shows no improvement
  - $\eta < 0$ 
    - Mortality of assured lives is moving in an opposite direction

# $\eta$ , males



# $\eta$ , females



# Reduction Factors

- 2001 Valuation Basic Table (VBT)
  - Developed by the Individual Life Insurance Valuation Mortality Research Task Force, SoA
  - Mortality experience underlying the 1990-95 Basic Mortality Tables is projected to 2001

# Reduction Factors

- 2001 Valuation Basic Table (VBT)

$$RF^M(x, t+s) = \begin{cases} 1 & x < 45 \\ \left(1 - \frac{0.01(x-45)}{10}\right)^s & 45 \leq x < 55 \\ 0.99^s & 55 \leq x < 80 \\ \left(1 - \frac{0.01(90-x)}{5}\right)^s & 80 \leq x < 90 \\ 1 & x > 90 \end{cases}$$

# Reduction Factors

- 2001 Valuation Basic Table (VBT)

$$RF^F(x, t+s) = \begin{cases} 1 & x < 45 \\ \left(1 - \frac{0.005(x-45)}{10}\right)^s & 45 \leq x < 55 \\ 0.995^s & 55 \leq x < 85 \\ \left(1 - \frac{0.005(90-x)}{5}\right)^s & 85 \leq x < 90 \\ 1 & x > 90 \end{cases}$$

# Reduction Factors

- Continuous Mortality Investigation Bureau (CMIB), IoA
- "92" series (CMIB, 1999):

$$RF(x, t + s) = \alpha(x) + [1 - \alpha(x)][1 - f_{20}(x)]^{s/20}$$

$$\alpha(x) = \left\{ \begin{array}{ll} 0.13 & x < 60 \\ 1 + 0.87 \frac{(x - 110)}{100} & 60 \leq x \leq 110 \\ 1 & x > 110 \end{array} \right\}$$

# Reduction Factors

- Continuous Mortality Investigation Bureau (CMIB), IoA
- "92" series (CMIB, 1999):

$$f_{20}(x) = \left\{ \begin{array}{ll} 0.55 & x < 60 \\ \frac{0.55(110 - x) + 0.29(x - 60)}{50} & 60 \leq x \leq 110 \\ 0.29 & x > 110 \end{array} \right\}$$

# Reduction Factors

- The Lee-Carter on Canadian Population
  - Recall that:

$$\begin{aligned}\ln(m_{x,t+s}) &= a_x + b_x k_{t+s} \\ &= a_x + b_x k_t + b_x (k_{t+s} - k_t) \\ &= \ln(m_{x,t}) + b_x (k_{t+s} - k_t)\end{aligned}$$

– And that  $k_t = c + k_{t-1} + \xi_{x,t}$

# Reduction Factors

- The Lee-Carter on Canadian Population
  - Then,

$$RF(x, t + s) = \frac{m_{x,t+s}}{m_t} = \exp(b_x (k_{t+s} - k_t))$$

- If  $k$  follows  $ARIMA(0,1,0)$

$$\begin{aligned} RF(x, t + s) &= \exp(b_x cs) \\ &= \exp(w_x s) \end{aligned}$$

# Reduction Factors

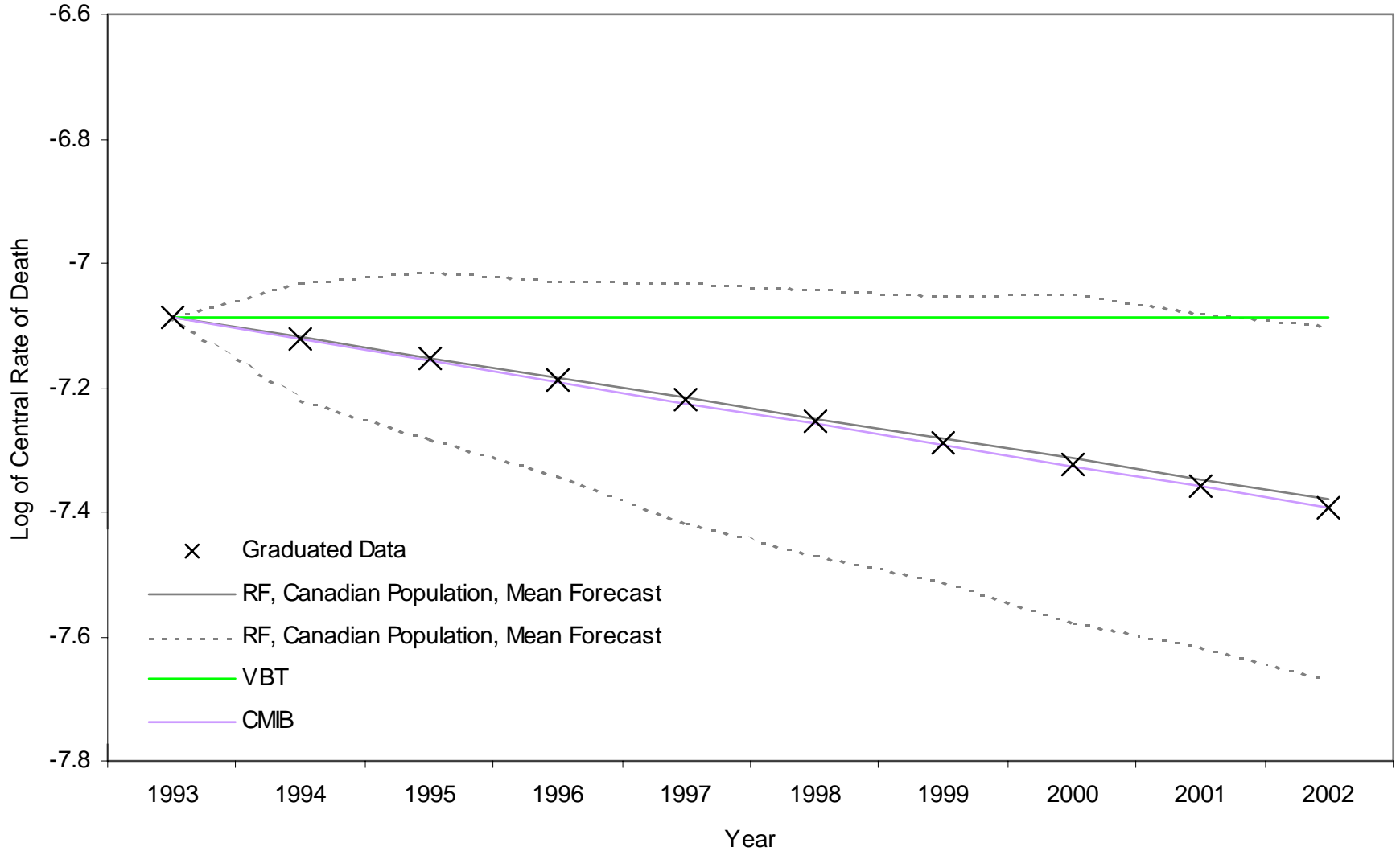
- The Lee-Carter on Canadian Population
  - Recall that under the “Brass-type” model:

$$\ln(m_{x,t}^{assured}) = \gamma_x + \eta_x \ln(m_{x,t}^{population})$$

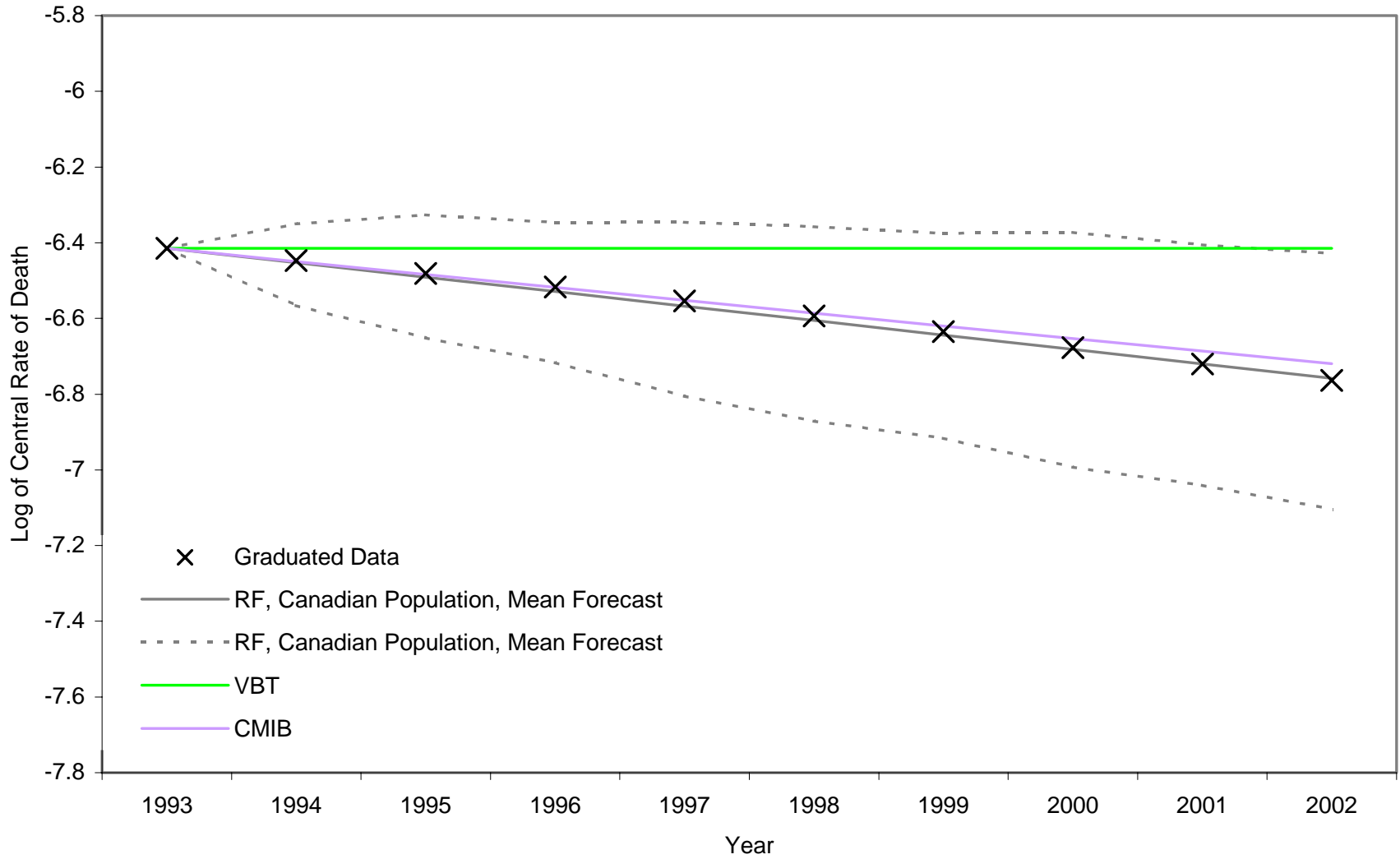
- It follows that

$$\begin{aligned} RF^{assured}(x, t + s) &= [RF^{population}(x, t + s)]^{\eta_x} \\ &= \exp(\eta_x w_x s) \end{aligned}$$

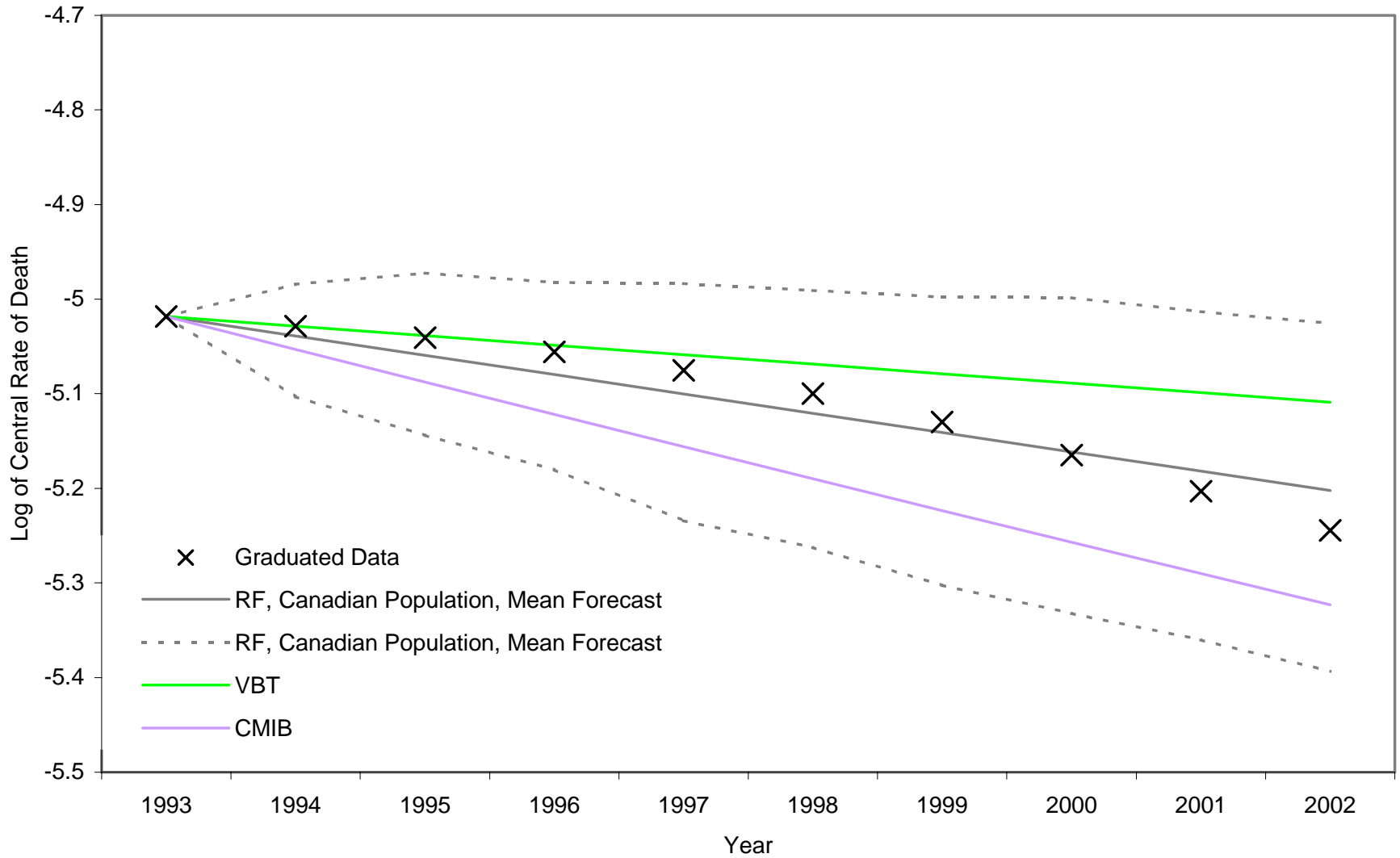
# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 16, Male



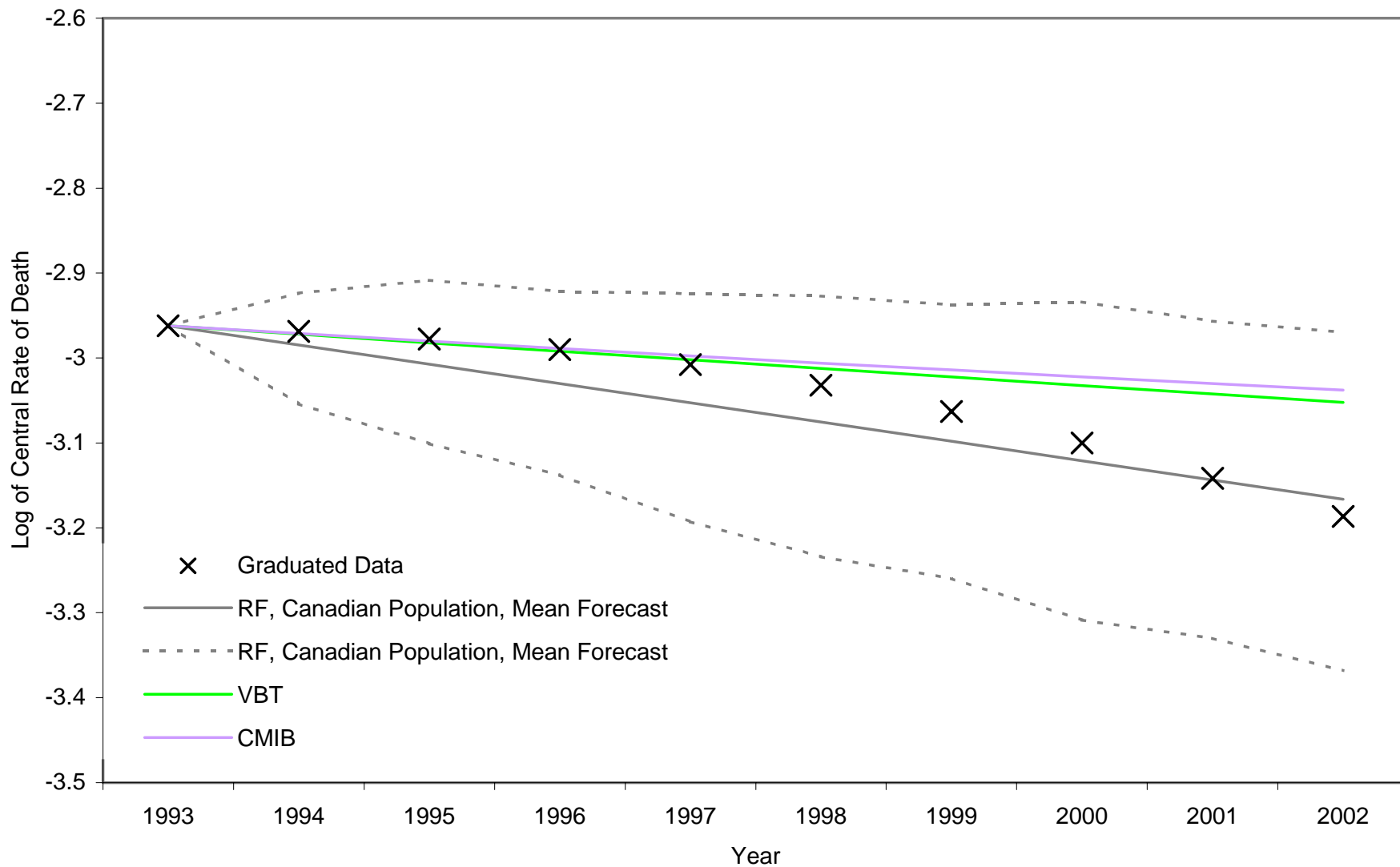
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 36, Male



### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 56, Male

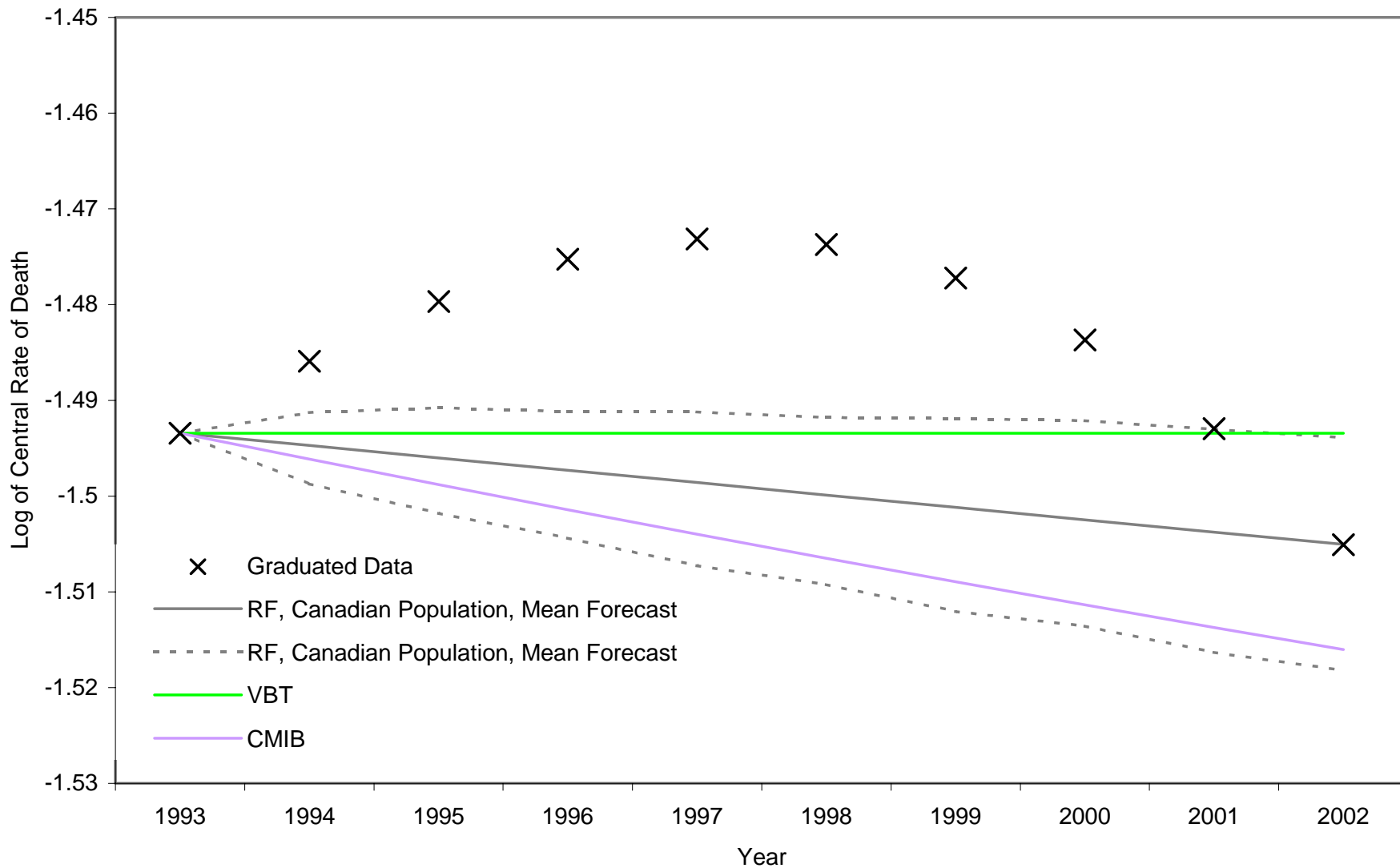


### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 76, Male

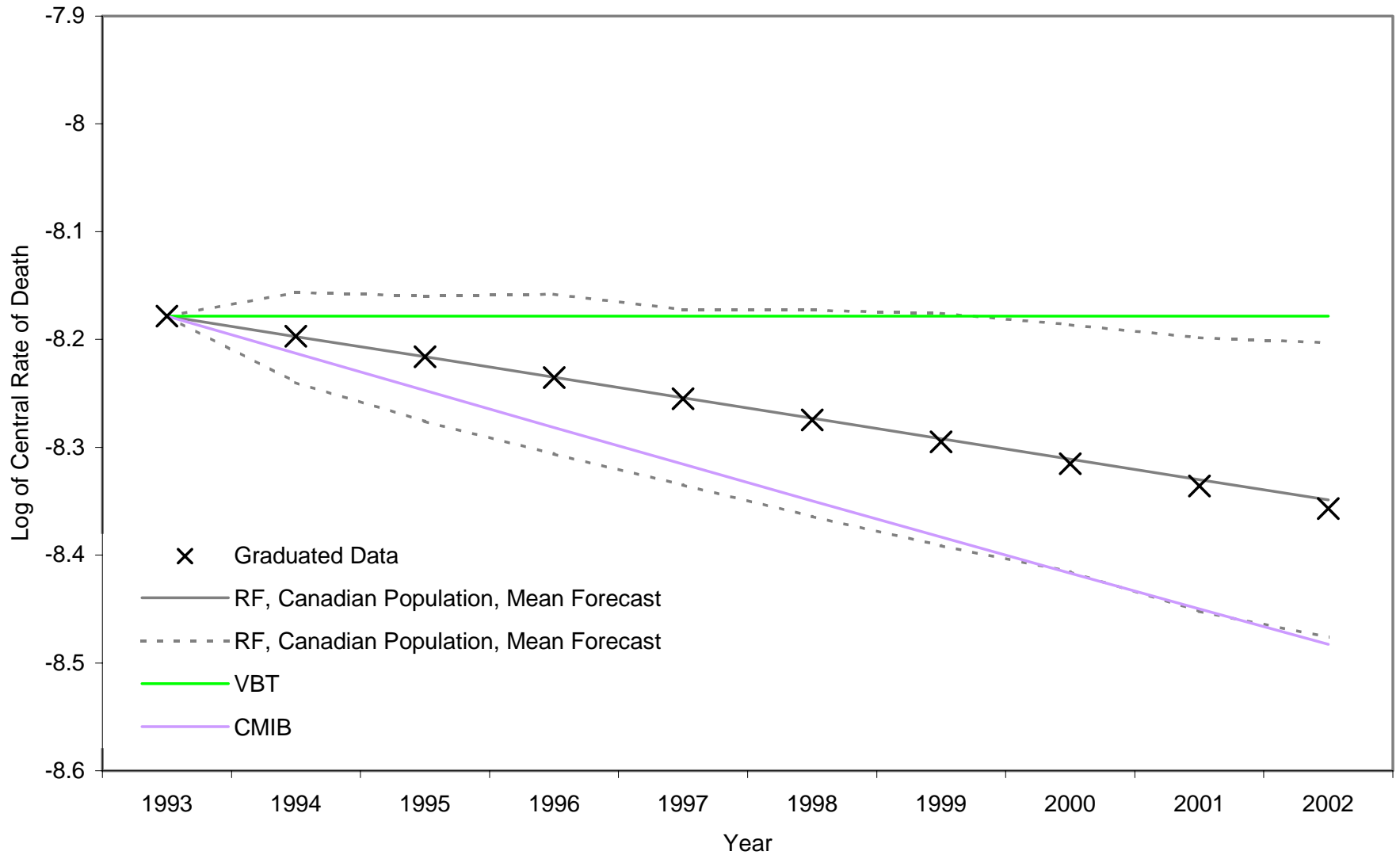


# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience

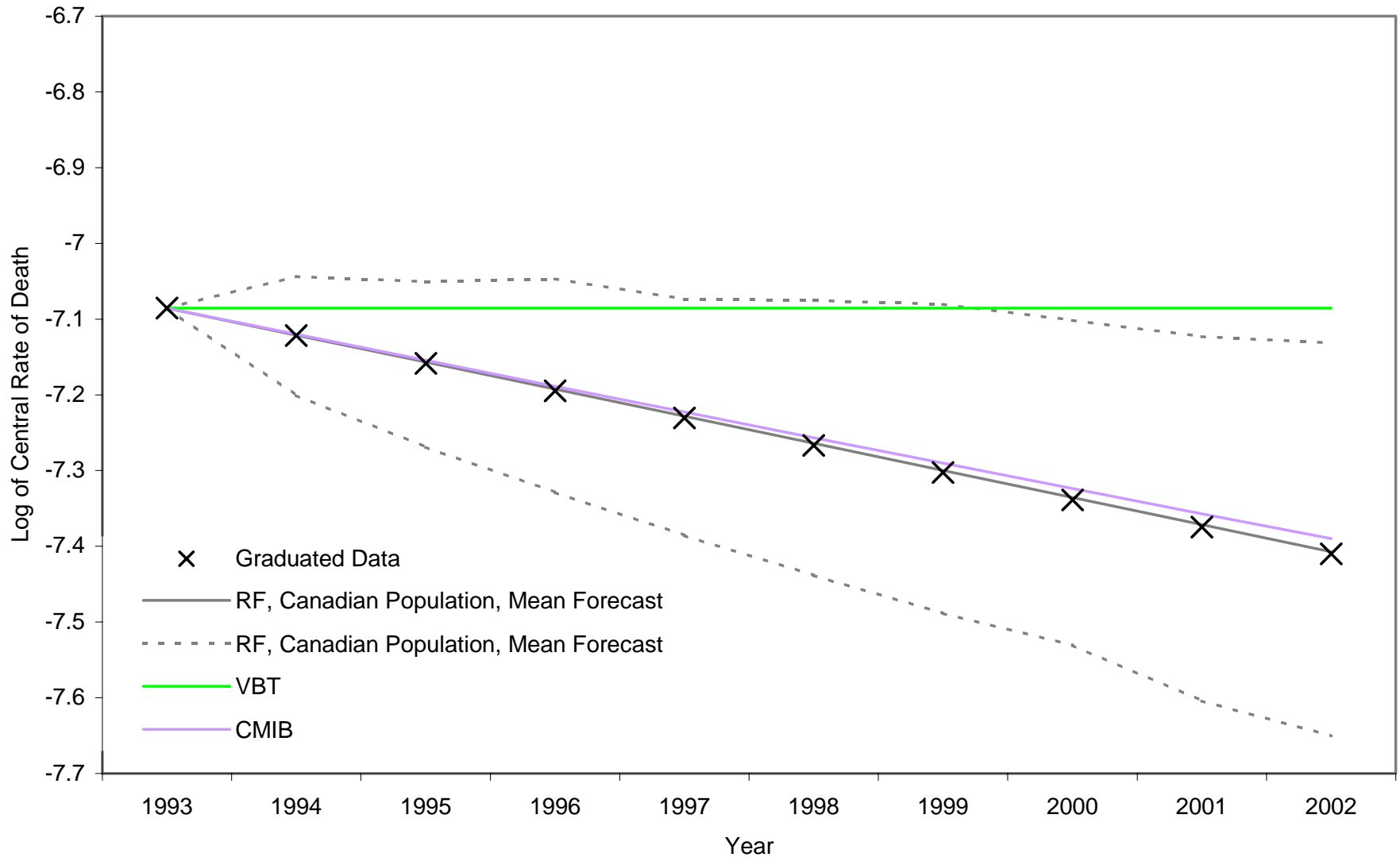
## Age 96, Male



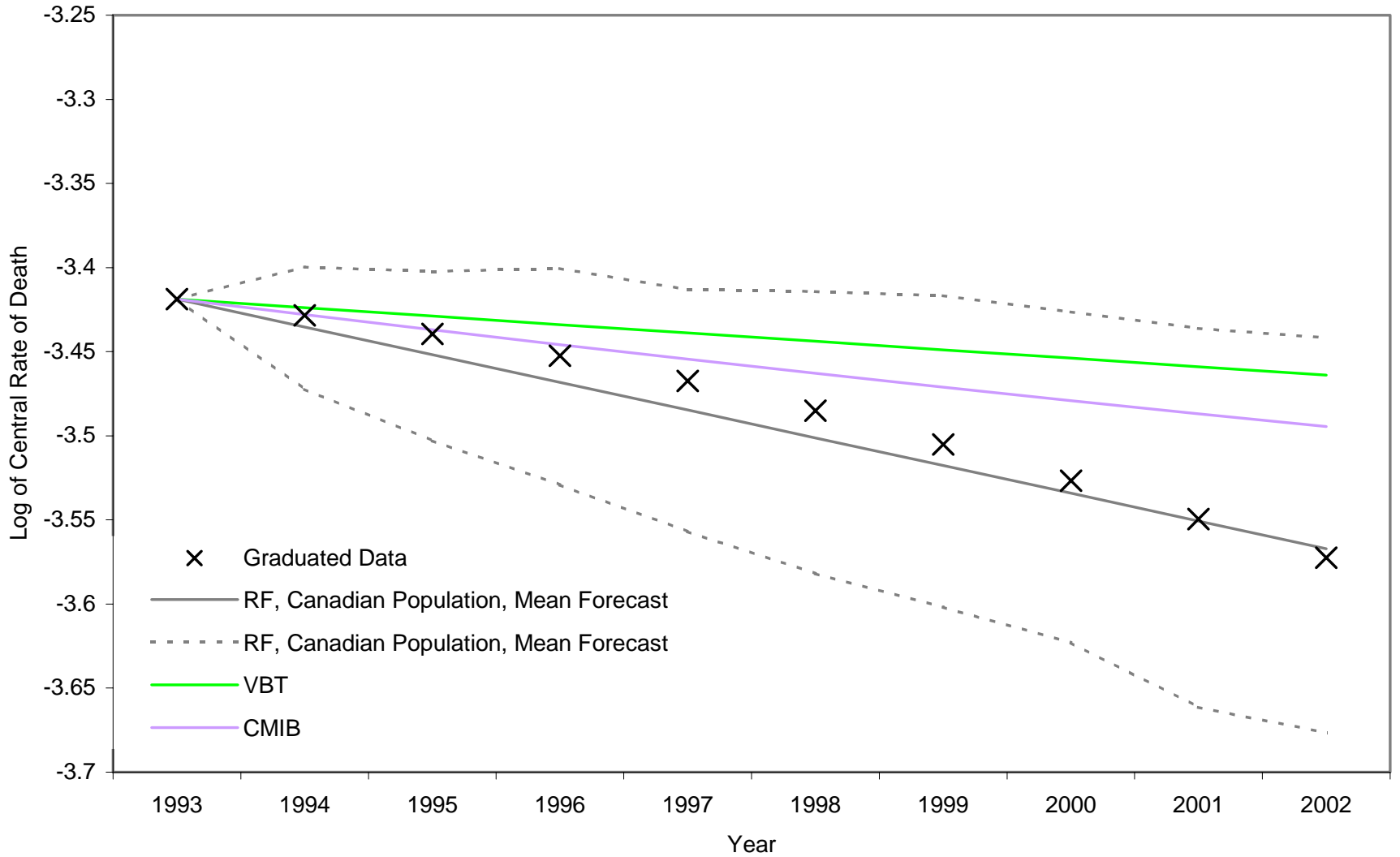
# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 16, Female



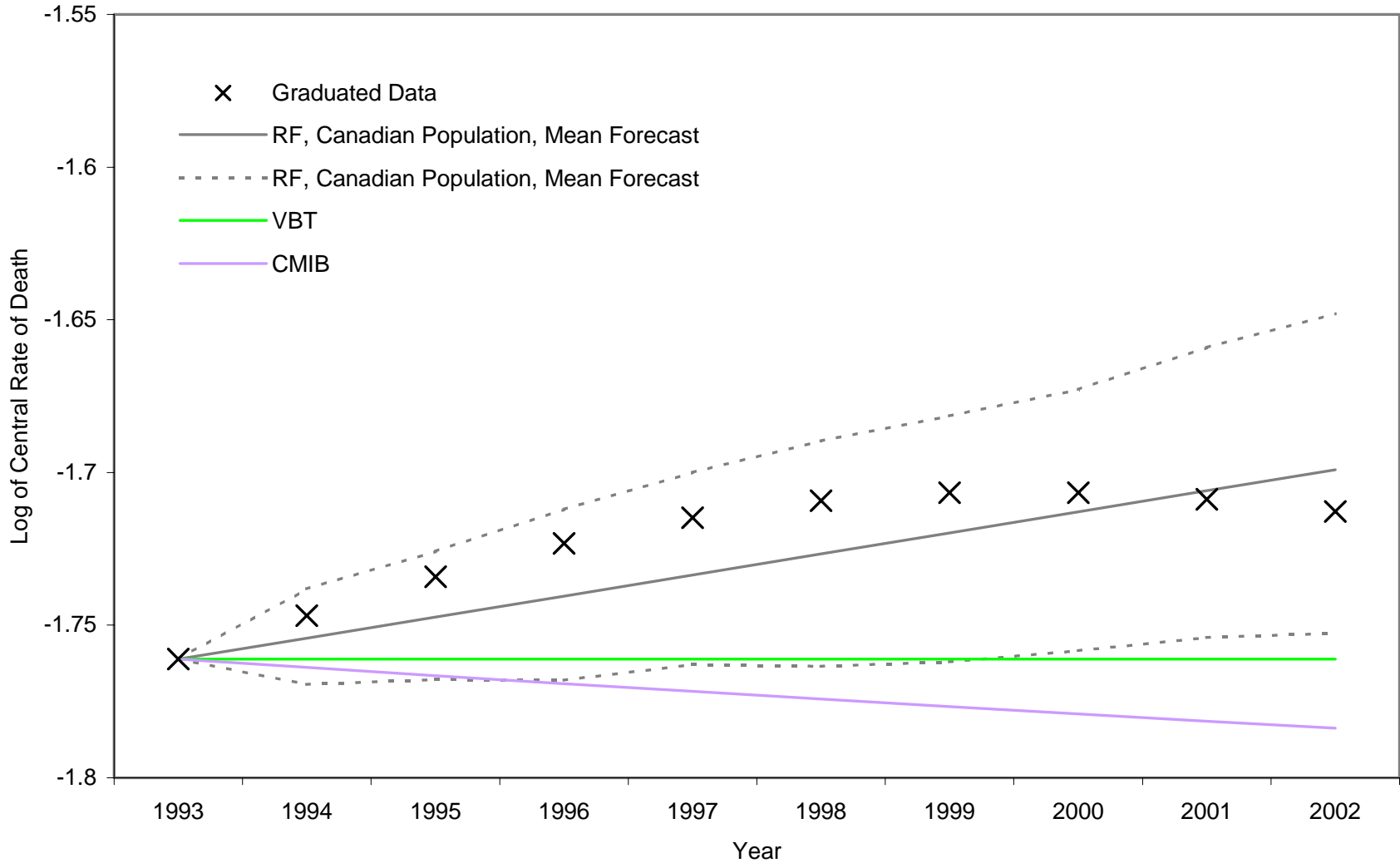
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 36, Female



### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 76, Female



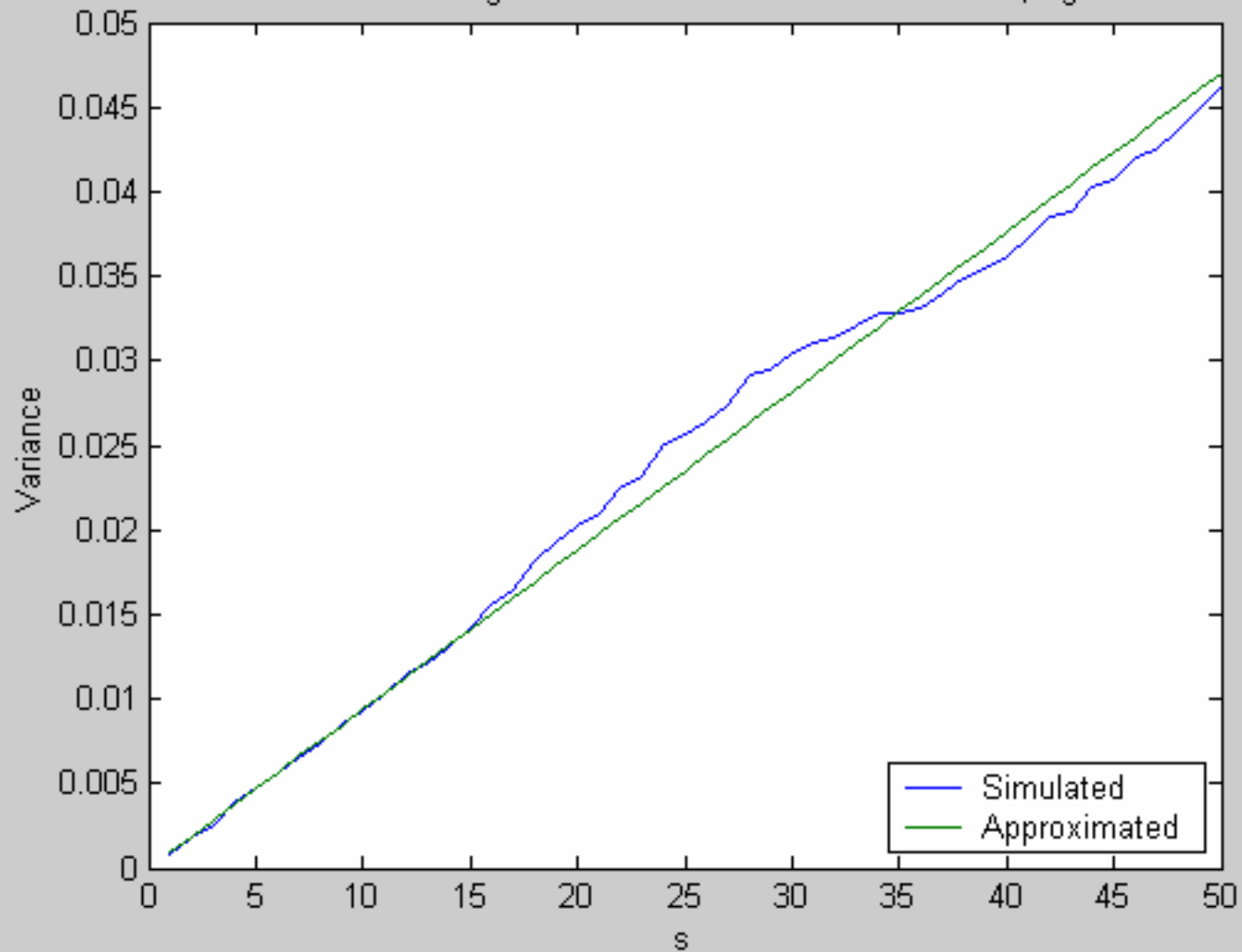
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 96, Female



# Interval Forecasts

- No analytical solution to the interval forecast of the reduction factors
- An approximate interval forecast is available when additional assumptions are imposed
- Empirically, the variance of the logarithm of the reduction factor is a (fairly) linear function of time
- The variance at  $s = 0$  is 0

Variance of the Logarithm of Reduction Factor Vs. Time, Age 56



# Interval Forecasts

- We write

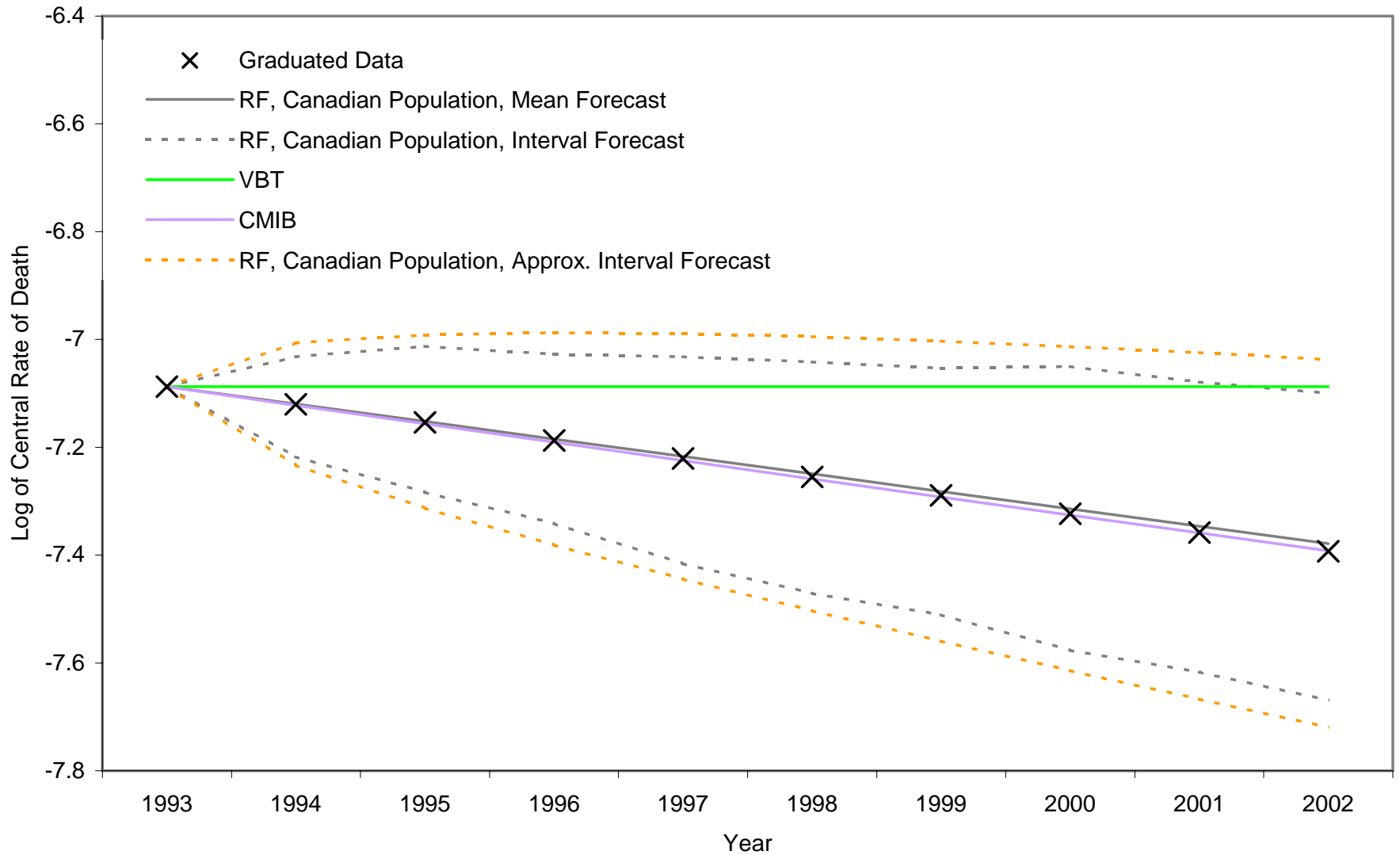
$$\text{Var} \left\{ \ln \left( RF^{population} (x, t + s) \right) \right\} = v_x s$$

- Assuming normality, it follows that

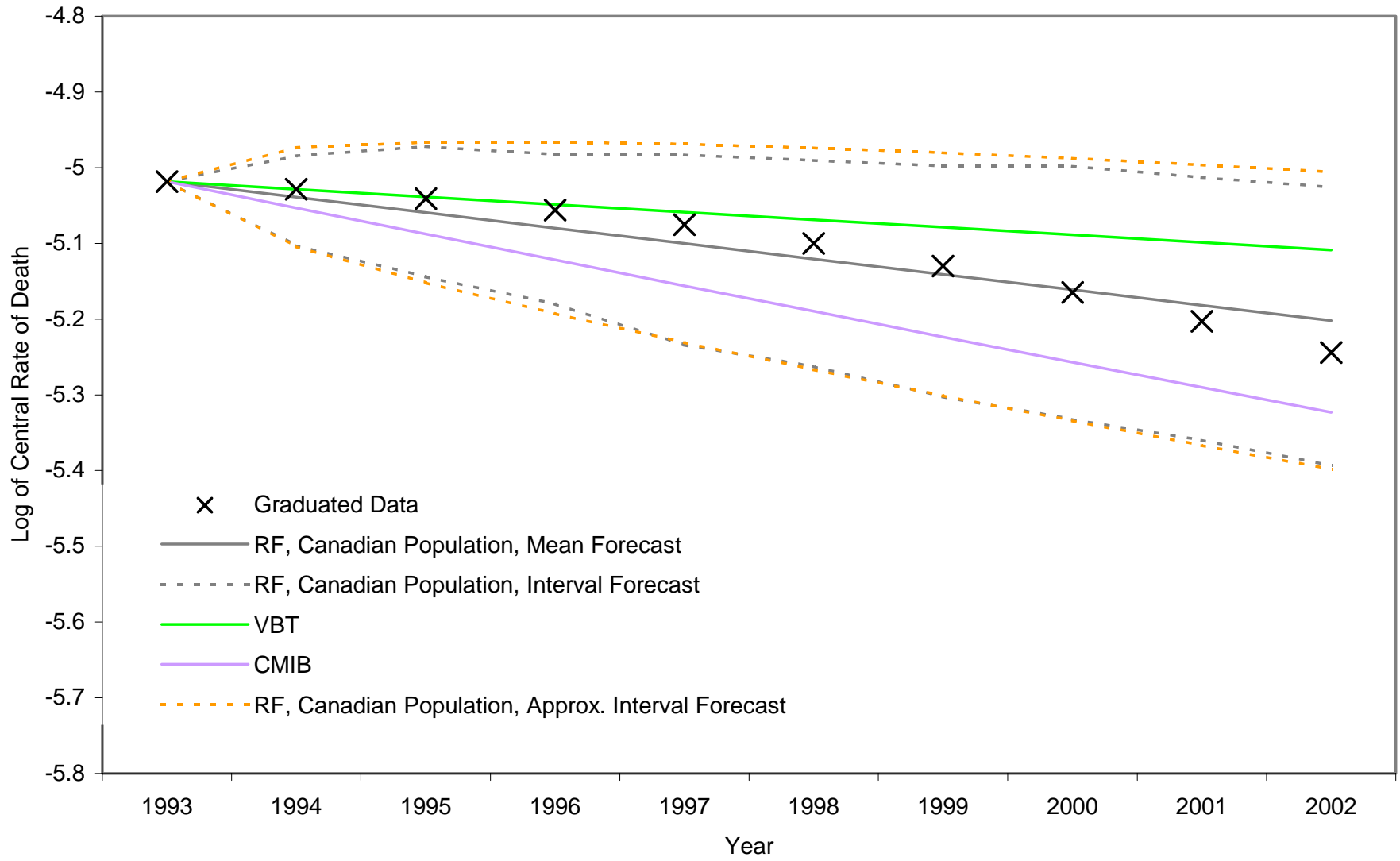
$$RF^{population, upper} (x, t + s) = \exp \left( \eta_x \left( w_x s + 2\sqrt{v_x s} \right) \right)$$

$$RF^{population, lower} (x, t + s) = \exp \left( \eta_x \left( w_x s - 2\sqrt{v_x s} \right) \right)$$

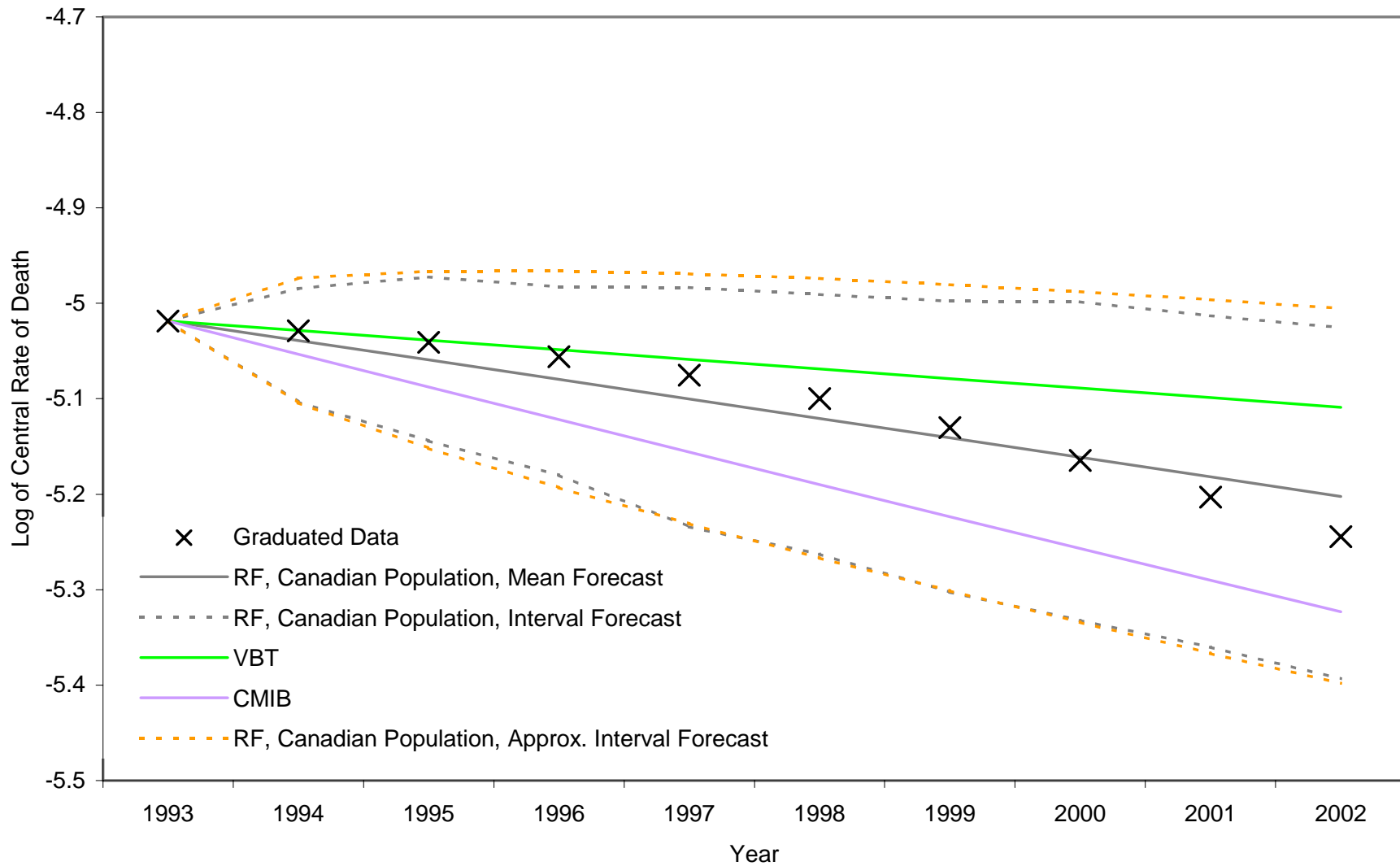
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 16, Male



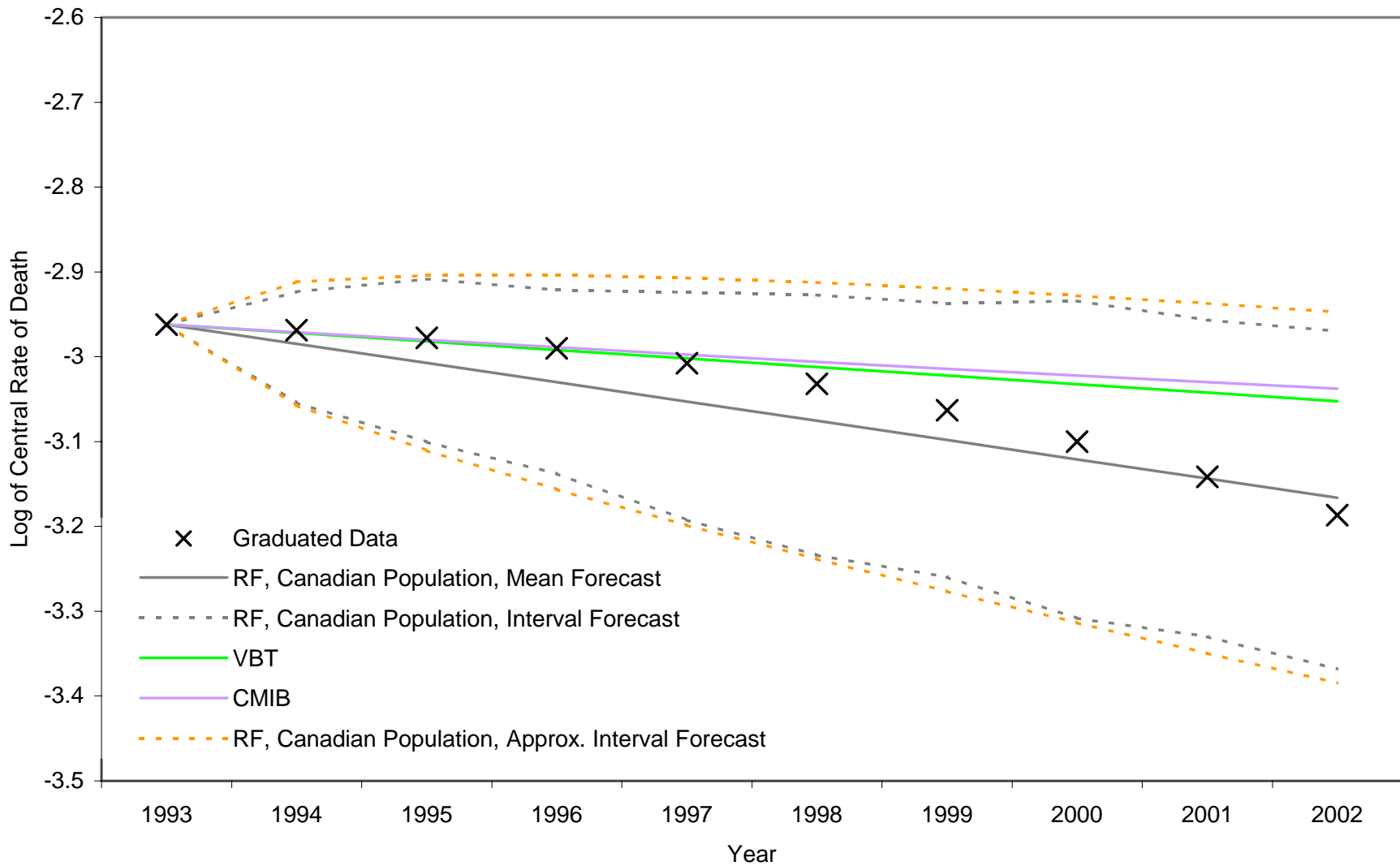
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 36, Male



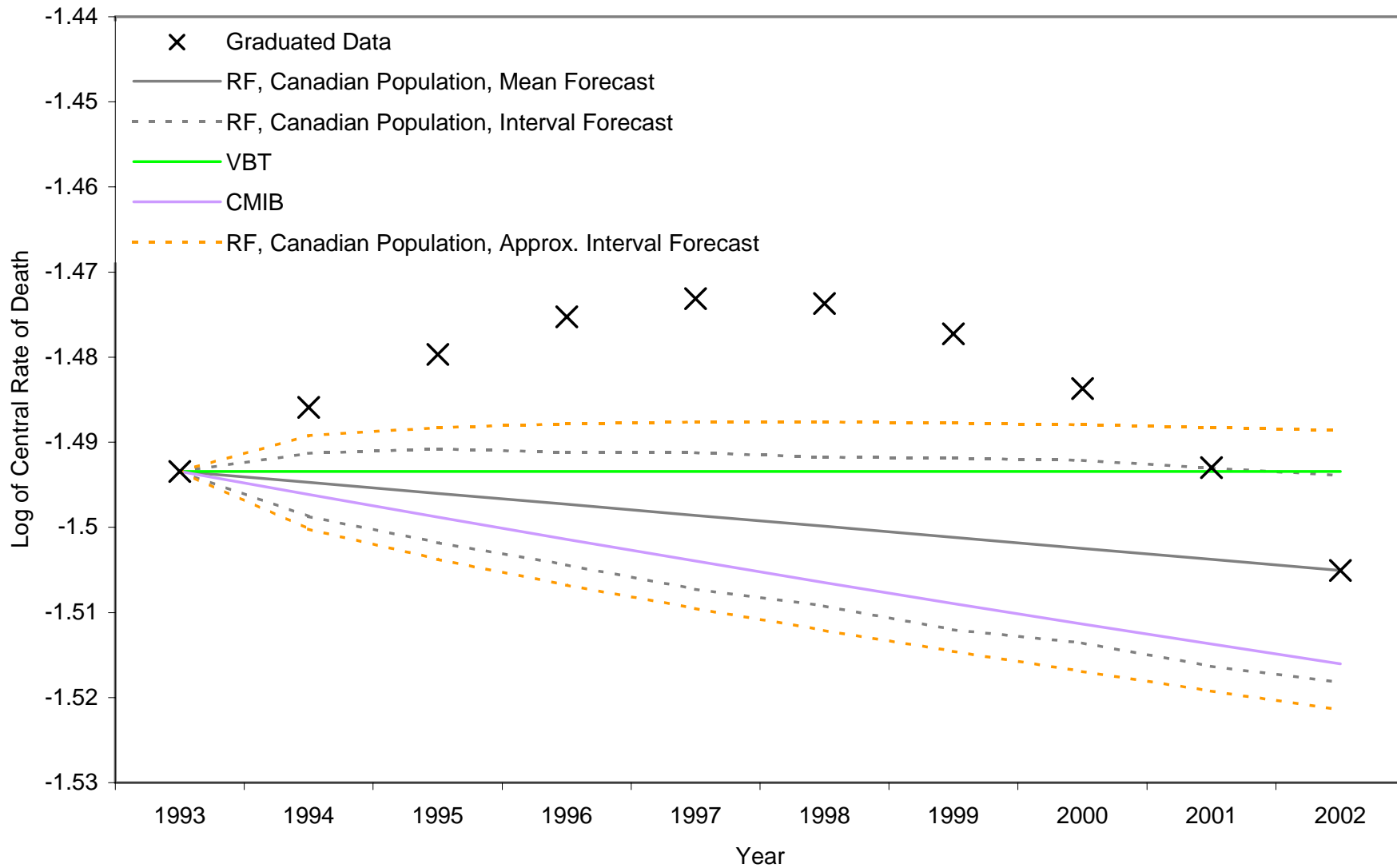
## Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 56, Male



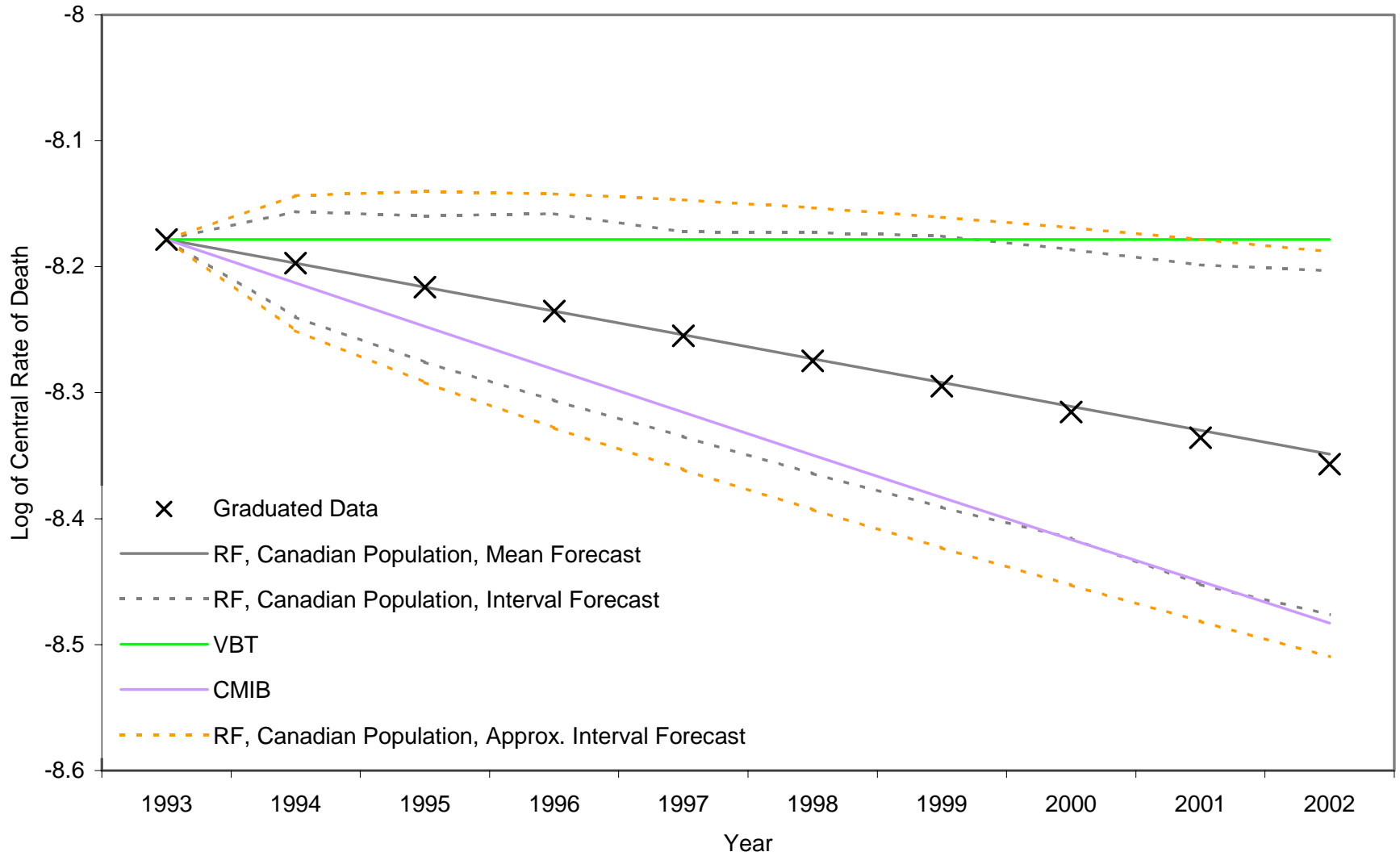
## Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 76, Male



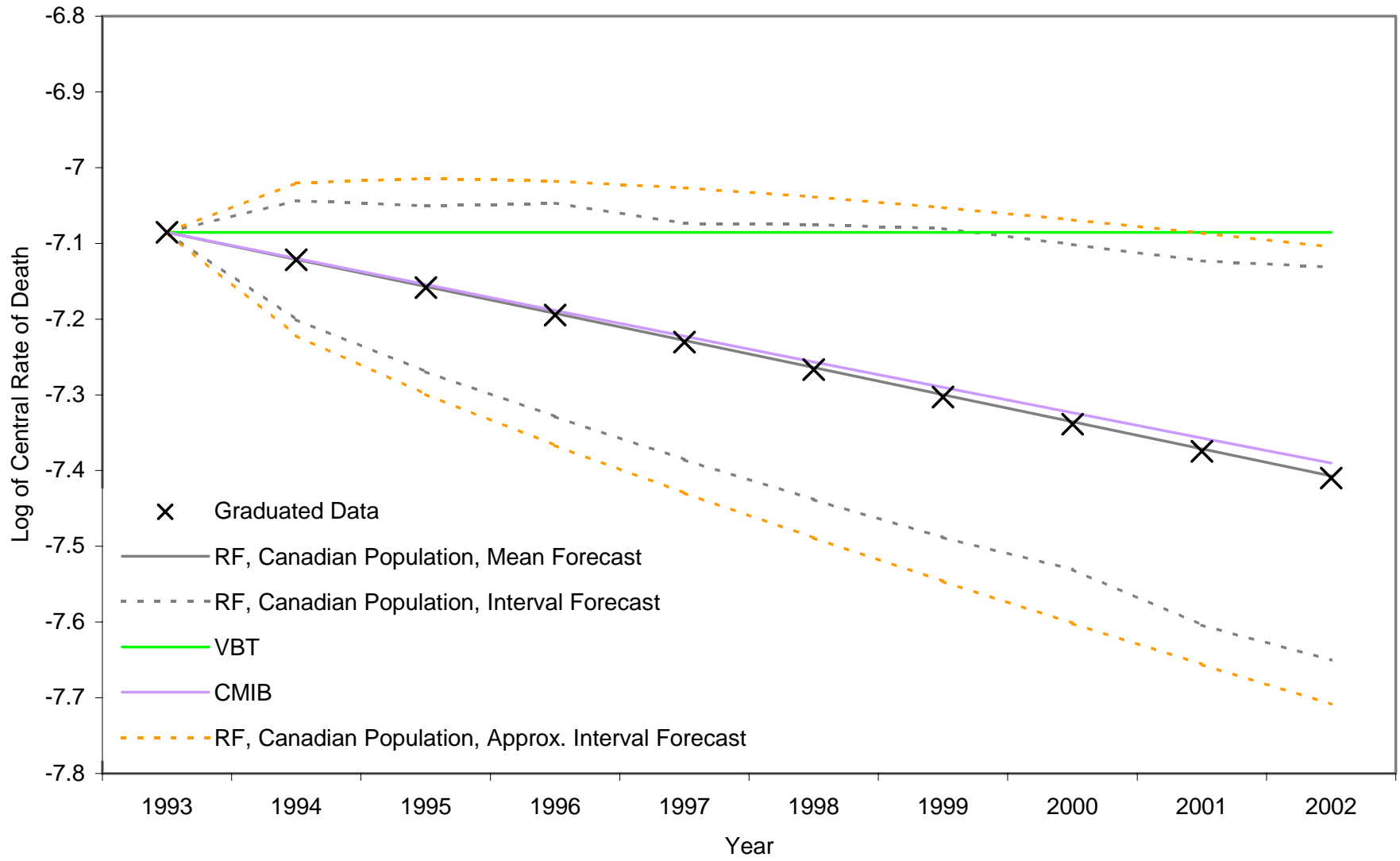
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 96, Male



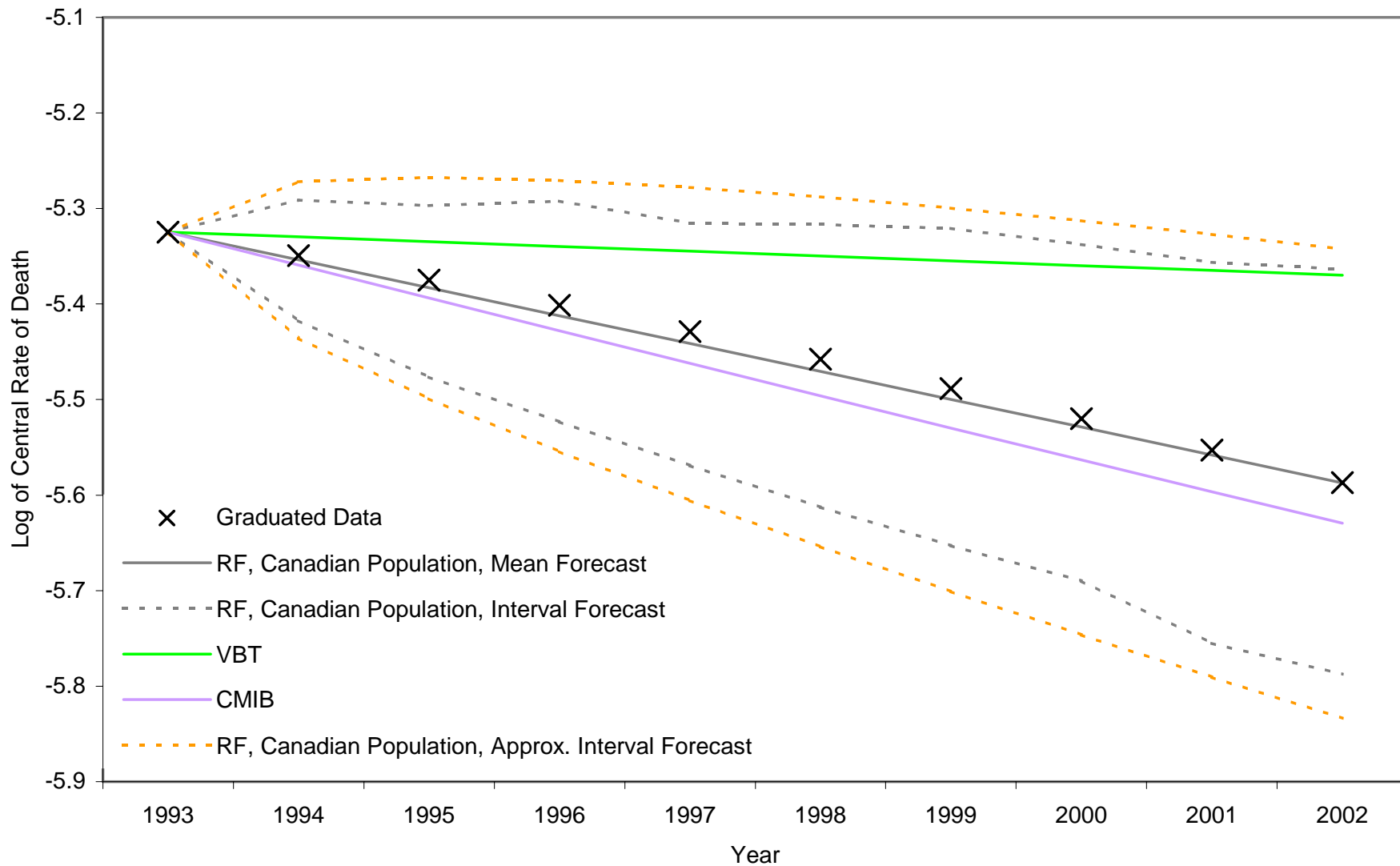
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 16, Female



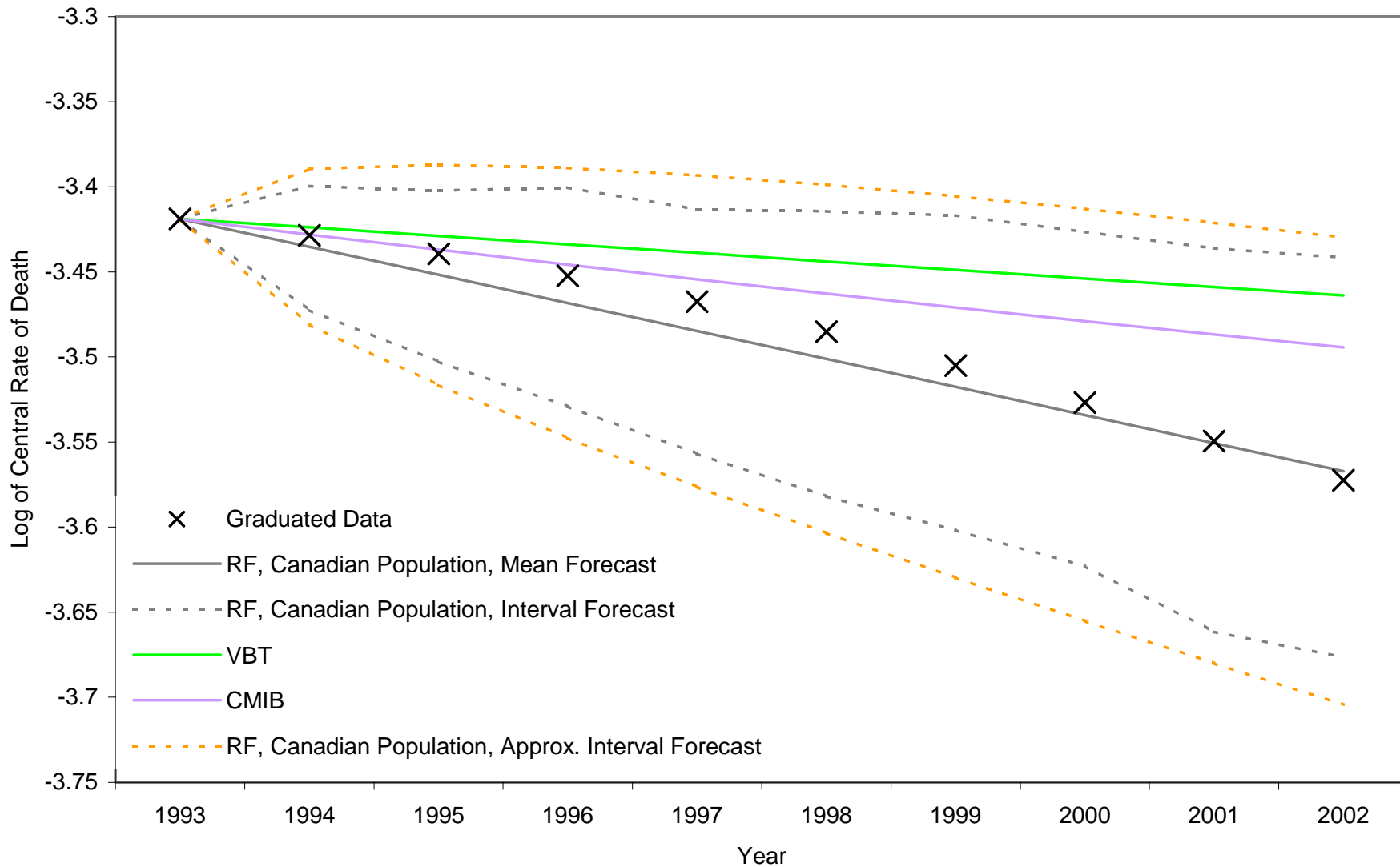
# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 36, Female



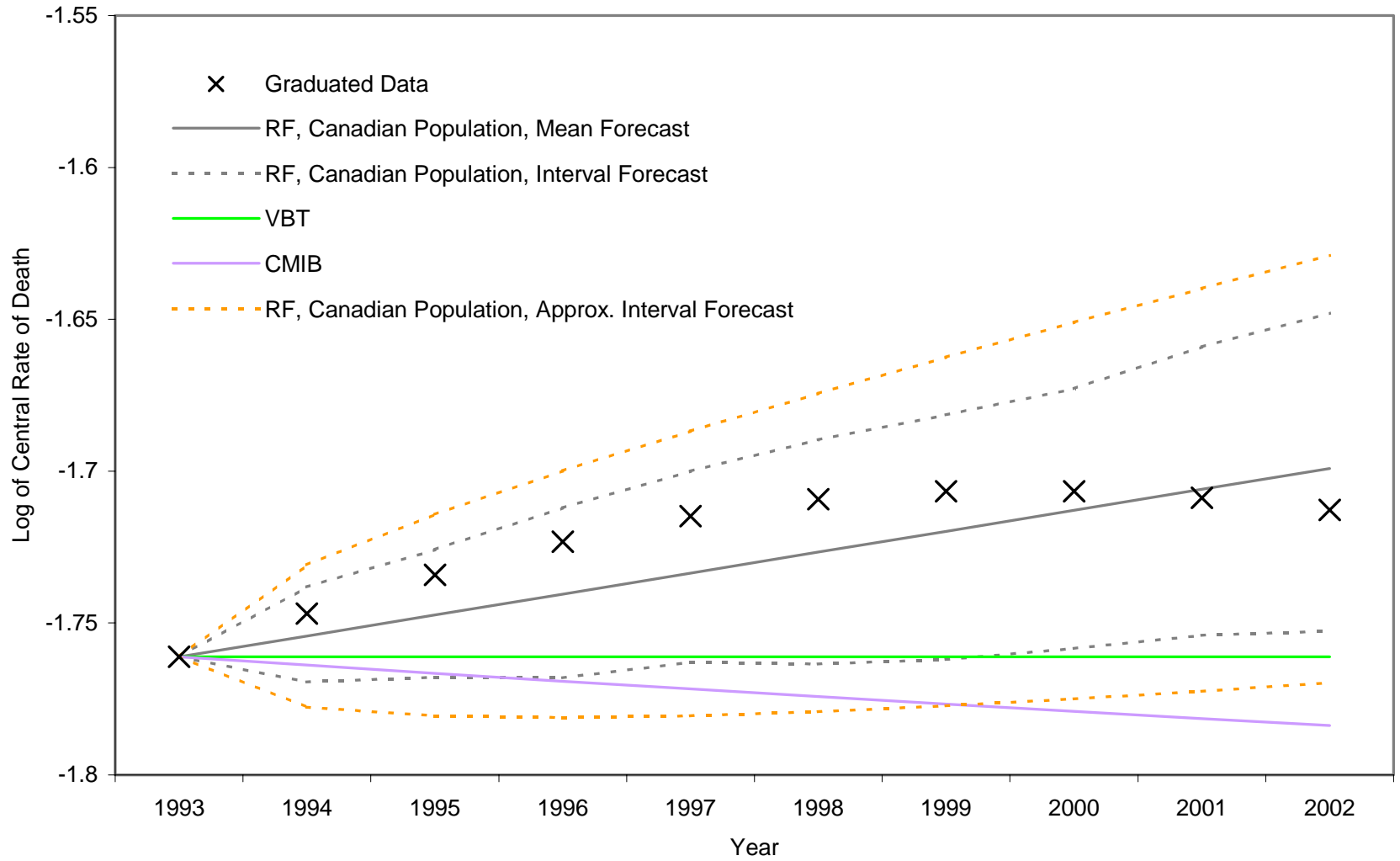
# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 56, Female



# Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 76, Female



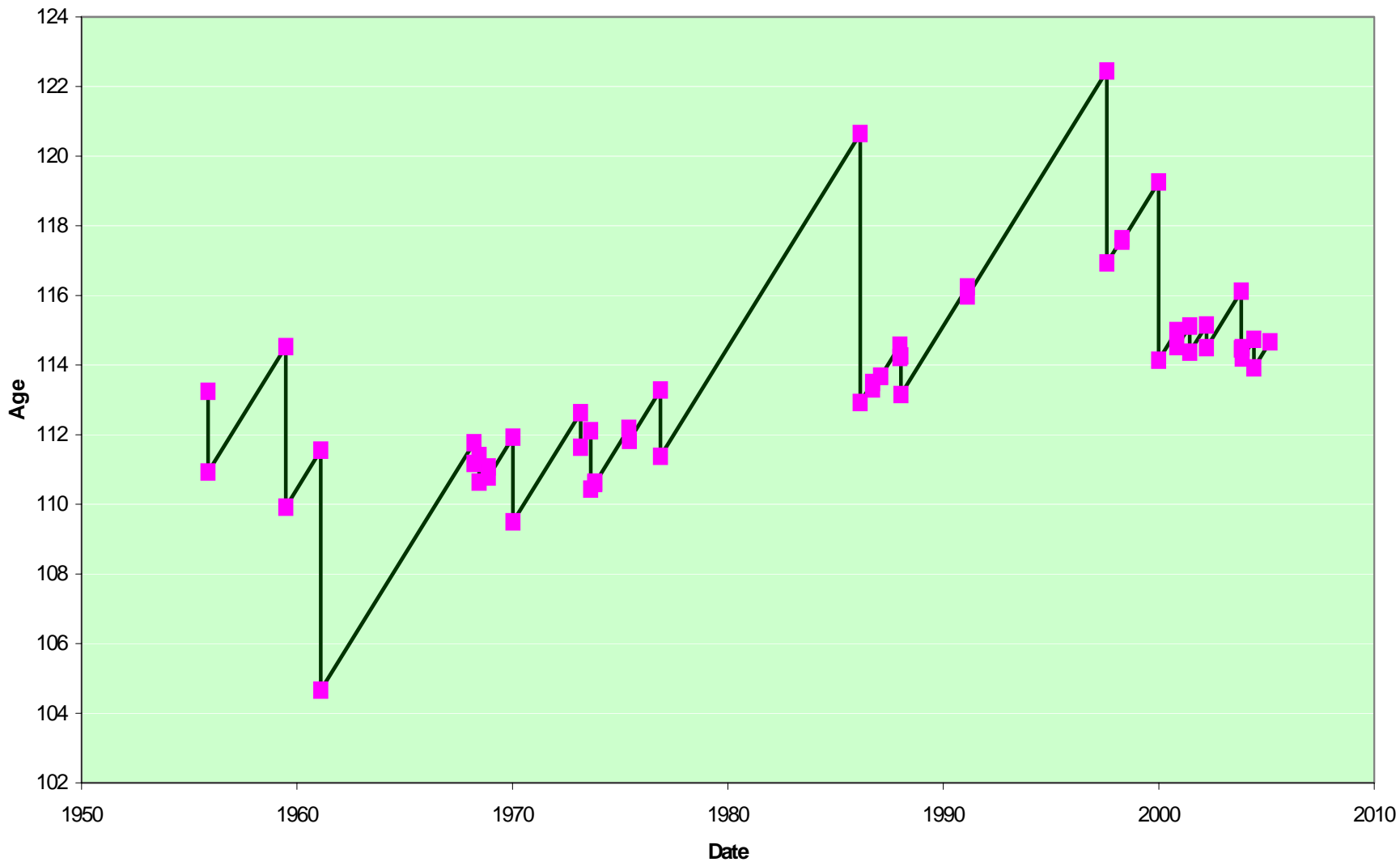
### Application of Mortality Reduction Factor (RF) on Canadian Assured Lives Mortality Experience Age 96, Female



# Remaining Work and Issues

- Length of projection
- Smoothing parameters
- Possible older and younger age adjustments
- Validation with more data from insured lives
- Comparison with annuitants mortality study
- Maximum life span issues
  - Is it increasing?
    - Fries – no
    - Wilmoth -- yes

# Age of oldest living person



# Remaining Work and Issues

- Mfad implications
- Parameter uncertainty
- Different models for  $k_t$
- Model uncertainty

# Some references

- Lee, R.D. & Carter, L. (1992). Modeling and forecasting the time series of U.S. mortality. *Journal of the American Statistical Association* **87**.
- Currie, I.D., Durban, M. & Eilers, P.H.C. (2004). Smoothing and forecasting mortality rates. *Statistical Modelling* **4**.
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